Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option

Take the Quick Check below. Refer to the Quick Review for help.

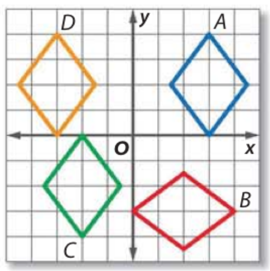
QuickCheck

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

1. *A* to *B*

2. *D* to *A*

3. *A* to *C*



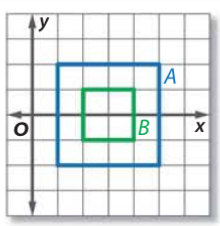
Find the sum of each pair of vectors.

4. $\langle 13, -4 \rangle + \langle -11, 9 \rangle$

5. $\langle 6, -31 \rangle + \langle -22, 3 \rangle$

6. **BAND** During part of a song, the drummer in a marching band moves from (1, 4) to (5, 1). Write the component form of the vector that describes his movement.

7. Determine whether the dilation from *A* to *B* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

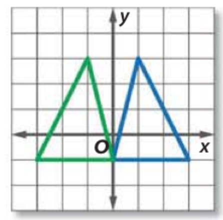


8. **PLAYS** Bob is making a model of an ant for a play. Find the scale factor of the model if the ant is $\frac{1}{2}$ inch long and the model is 1 foot long.

QuickReview

Example 1

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.



Each vertex and its image are the same distance from the *y*-axis. This is a reflection.

Example 2

Write the component form of $\overrightarrow{AB}$ for $A(-1, 1)$ and $B(4, -3)$.

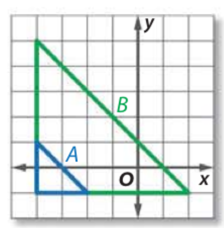
$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$ Component form of vector

$= \langle 4 - (-1), -3 - 1 \rangle$ Substitute.

$= \langle 5, -4 \rangle$ Simplify.

Example 3

Determine whether the dilation from *A* to *B* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.



B is larger than *A*, so it is an enlargement.

The distance between the vertices of ***A*** is **2** and the corresponding distance for ***B*** is **6**.

The scale factor is $\frac{6}{2}$ or 3.

2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

Get Started on the Chapter

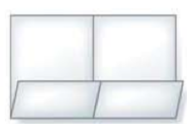
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 9. To get ready, identify important terms and organize your resources. You may refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

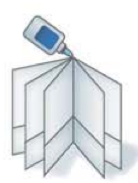
Transformations and Symmetry Make this Foldable to help you organize your Chapter 9 notes about transformations and symmetry. Begin with three sheets of notebook paper.

1. **Fold** each sheet of paper in half.

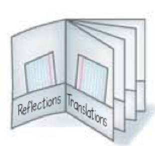
2. **Open** the folded papers and fold each paper lengthwise two inches, to form a pocket.



3. **Glue** the sheets side-by-side to create a booklet.



4. **Label** each of the pockets as shown.



NewVocabulary

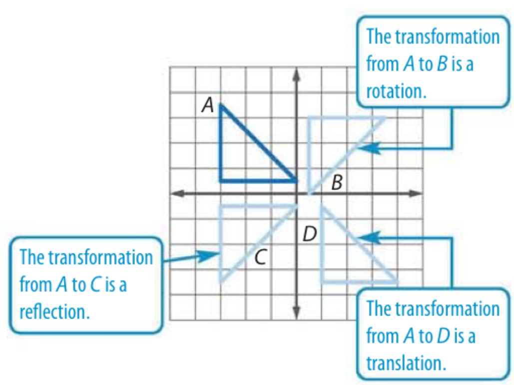
English		Español
line of reflection	p. 623	línea de reflexión
center of rotation	p. 640	centro de rotación
angle of rotation	p. 640	ángulo de rotación
composition of transformations	p. 651	composición de transformaciones
symmetry	p. 663	símetria
line symmetry	p. 663	símetria lineal
line of symmetry	p. 663	eje de símetria

ReviewVocabulary

reflection reflexión a transformation representing a flip of the figure over a point, line or plane

rotation rotación a transformation that turns every point of a preimage through a specified angle and direction about a fixed point

translation traslación a transformation that moves all points of a figure the same distance in the same direction



LESSON 9-1 Reflections

Then	Now	Why?
You identified reflections and verified them as congruence transformations.	1 Draw reflections. 2 Draw reflections in the coordinate plane.	Notice in this water reflection that the distance a point lies above the water line appears the same as the distance its image lies below the water.

NewVocabulary
line of reflection

Common Core State Standards

Content Standards

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematical Practices

5 Use appropriate tools strategically.

7 Look for and make use of structure.

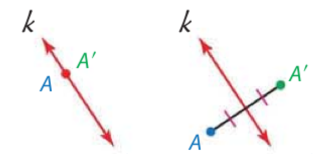
1 Draw Reflections

In Lesson 4-7, you learned that a reflection or *flip* is a transformation in a line called the **line of reflection**. Each point of the preimage and its corresponding point on the image are the same distance from this line.

KeyConcept Reflection in a Line

A reflection in a line is a function that maps a point to its image such that

- if the point is on the line, then the image and preimage are the same point, or
- if the point is not on the line, the line is the perpendicular bisector of the segment joining the two points.



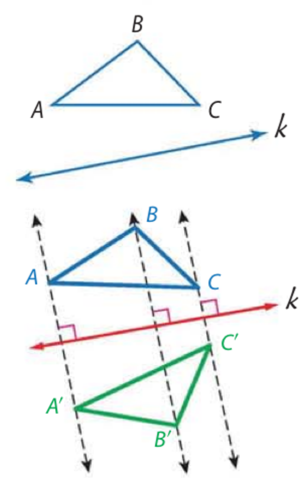
A is on line *k*. *A* is not on line *k*.

A′, *A″*, *A‴*, and so on, name corresponding points for one or more transformations.

To reflect a polygon in a line, reflect each of the polygon's vertices. Then connect these vertices to form the reflected image.

Example 1 Reflect a Figure in a Line

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.



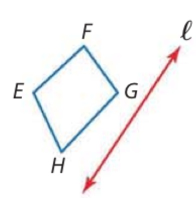
Step 1 Draw a line through each vertex that is perpendicular to line *k*.

Step 2 Measure the distance from point *A* to line *k*. Then locate *A′* the same distance from line *k* on the opposite side

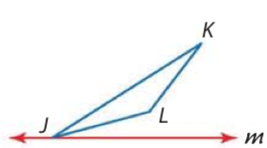
Step 3 Repeat Step 2 to locate points *B′* and *C′*. Then connect vertices *A′*, *B′*, and *C′* to form the reflected image.

GuidedPractice

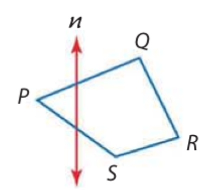
1A.



1B.



1C.



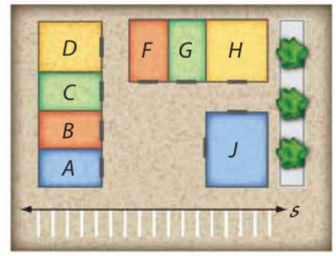
Real-World Career

Photographer Photographers take photos for a variety of reasons such as journalism, art, to record an event, or for scientific purposes. In some photography fields such as photojournalism and scientific photography, a bachelor's degree is required. For others, such as portrait photography, technical proficiency is the only requirement.

Recall that a reflection is a *congruence transformation* or *isometry.* In the figure in Example 1, $\triangle ABC \cong \triangle A'B'C'$.

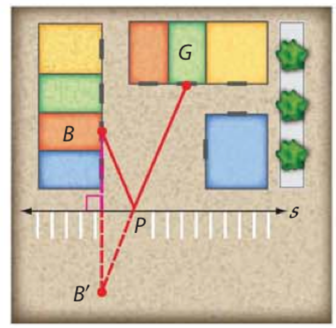
Real-World Example 2 Minimize Distance by Using a Reflection

SHOPPING Suppose you are going to buy clothes in Store B, return to your car, and then buy shoes at Store G. Where along line s of parking spaces should you park to minimize the distance you will walk?



Understand You are asked to locate a point P on line s such that $BP + PG$ has the least possible value.

Plan The total distance from B to P and then from P to G is least when these three points are collinear. Use the reflection of point B in line s to find the location for point P.

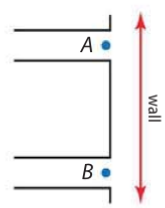


Solve Draw $\overline{B'G}$. Locate P at the intersection of line s and $\overline{B'G}$.

Check Compare the sum $BP + PG$ for each case to verify that the location found for P minimizes this sum.

Guided Practice

2. TICKET SALES Joy wants to select a good location to sell tickets for a dance. Locate point P such that the distance someone would have to walk from Hallway A, to point P on the wall, and then to their next class in Hallway B is minimized.

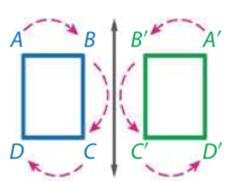


2 Draw Reflections in the Coordinate Plane

Reflections can also be performed in the coordinate plane by using the techniques presented in Example 3.

Study Tip

Characteristics of a Reflection Reflections, like all isometries, preserve distance, angle measure, betweenness of points, and collinearity. The orientation of a preimage and its image, however, are reversed.

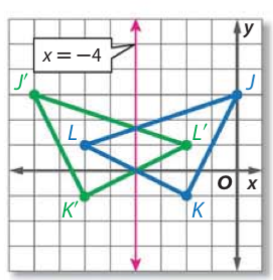


Example 3 Reflect a Figure in a Horizontal or Vertical Line

Triangle JKL has vertices $J(0, 3)$, $K(-2, -1)$, and $L(-6, 1)$. Graph $\triangle JKL$ and its image in the given line.

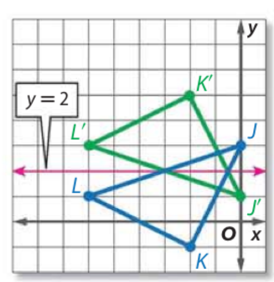
a. $x = -4$

Find a corresponding point for each vertex so that a vertex and its image are equidistant from the line $x = -4$.



b. $y = 2$

Find a corresponding point for each vertex so that a vertex and its image are equidistant from the line $y = 2$.



GuidedPractice

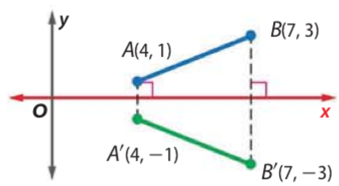
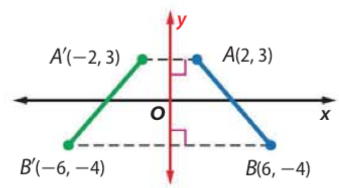
Trapezoid *RSTV* has vertices $R(-1, 1)$, $S(4, 1)$, $T(4, -1)$, and $V(-1, -3)$. Graph trapezoid *RSTV* and its image in the given line.

3A. $y = -3$

3B. $x = 2$

When the line of reflection is the x- or y-axis, you can use the following rule.

KeyConcept Reflection in the x- or y-axis

	Reflection in the x-axis	Reflection in the y-axis
Words	To reflect a point in the x-axis, multiply its y-coordinate by -1.	To reflect a point in the y-axis, multiply its x-coordinate by -1.
Symbols	$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$
Example		

ReadingMath

Coordinate Function Notation The expression $P(a, b) \rightarrow P'(a, -b)$ can be read as *point P with coordinates a and b is mapped to new location P prime with coordinates a and negative b.*

Example 4 Reflect a Figure in the x- or y-axis

Graph each figure and its image under the given reflection.

a. **$\triangle ABC$ with vertices $A(-5, 3)$, $B(2, 0)$, and $C(1, 2)$ in the x-axis**

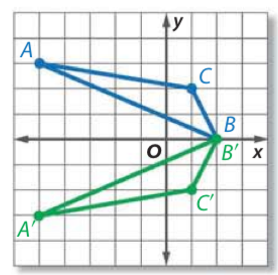
Multiply the y-coordinate of each vertex by -1.

$(x, y) \rightarrow (x, -y)$

$A(-5, 3) \rightarrow A'(-5, -3)$

$B(2, 0) \rightarrow B'(2, 0)$

$C(1, 2) \rightarrow C'(1, -2)$



b. **parallelogram *PQRS* with vertices $P(-4, 1)$, $Q(2, 3)$, $R(2, -1)$, and $S(-4, -3)$ in the y-axis**

Multiply the x-coordinate of each vertex by -1.

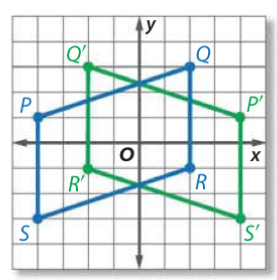
$(x, y) \rightarrow (-x, y)$

$P(-4, 1) \rightarrow P'(4, 1)$

$Q(2, 3) \rightarrow Q'(-2, 3)$

$R(2, -1) \rightarrow R'(-2, -1)$

$S(-4, -3) \rightarrow S'(4, -3)$



StudyTip

Invariant Points In Example 4a, point *B* is called an *invariant point* because it maps onto itself. Only points that lie on the line of reflection are invariant under a reflection.

GuidedPractice

4A. rectangle with vertices $E(-4, -1)$, $F(2, 2)$, $G(3, 0)$, and $H(-3, -3)$ in the x-axis

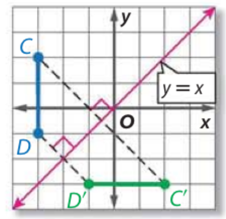
4B. $\triangle JKL$ with vertices $J(3, 2)$, $K(2, -2)$, and $L(4, -5)$ in the y-axis

ReviewVocabulary

Perpendicular Lines Two nonvertical lines are perpendicular if and only if the product of their slopes is −1.

You can also reflect an image in the line $y = x$.

The slope of $y = x$ is 1. In the graph shown, $\overline{CC'}$ is perpendicular to $y = x$, so its slope is −1. From $C(-3, 2)$, move right 2.5 units and down 2.5 units to reach $y = x$. From this point on $y = x$, move right 2.5 units and down 2.5 units to locate $C'(2, -3)$. Using a similar method, the image of $D(-3, -1)$ is found to be $D'(-1, -3)$.



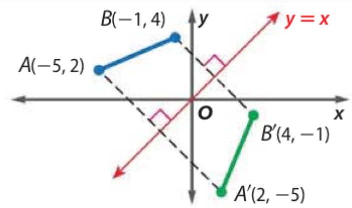
Comparing the coordinates of these and other examples leads to the following rule for reflections in the line $y = x$.

KeyConcept Reflection in Line $y = x$

Words To reflect a point in the line $y = x$, interchange the x- and y-coordinates.

Symbols $(x, y) \rightarrow (y, x)$

Example



Example 5 Reflect a Figure in the Line $y = x$

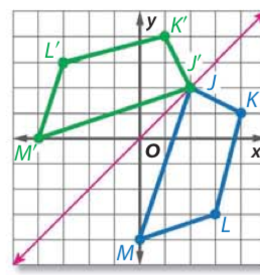
StudyTip

Preimage and Image In this book, the preimage will always be blue and the image will always be green.

Quadrilateral *JKLM* has vertices $J(2, 2)$, $K(4, 1)$, $L(3, -3)$, and $M(0, -4)$. Graph *JKLM* and its image $J'K'L'M'$ in the line $y = x$.

Interchange the x- and y-coordinates of each vertex.

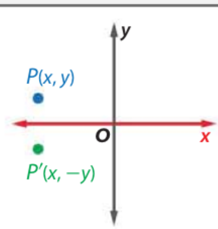
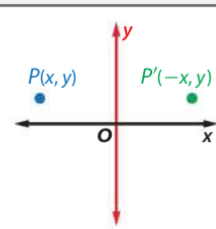
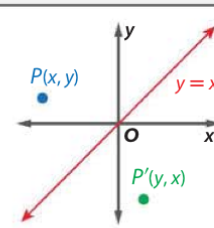
(x, y)	→	(y, x)
$J(2, 2)$	→	$J'(2, 2)$
$K(4, 1)$	→	$K'(1, 4)$
$L(3, -3)$	→	$L'(-3, 3)$
$M(0, -4)$	→	$M'(-4, 0)$



GuidedPractice

5. $\triangle BCD$ has vertices $B(-3, 3)$, $C(1, 4)$, and $D(-2, -4)$. Graph $\triangle BCD$ and its image in the line $y = x$.

ConceptSummary Reflection in the Coordinate Plane

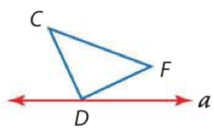
Reflection in the x-axis	Reflection in the y-axis	Reflection in the line $y = x$
		
$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$	$(x, y) \rightarrow (y, x)$

Check Your Understanding

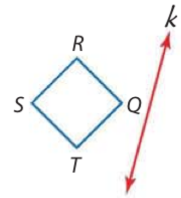
= Step-by-Step Solutions begin on page R14.

Example 1 **Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.**

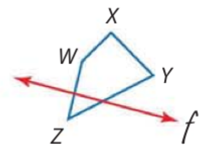
1.



2.

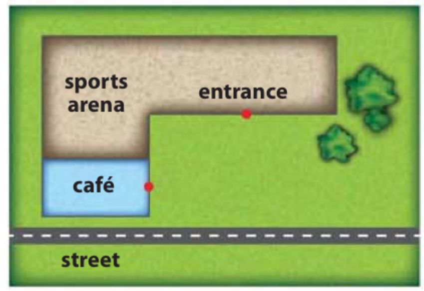


3.



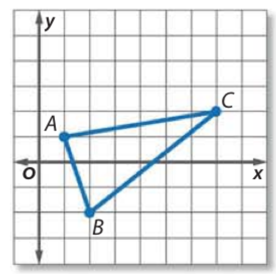
Example 2

4. **SPORTING EVENTS** Toru is waiting at a café for a friend to bring him a ticket to a sold-out sporting event. At what point P along the street should the friend try to stop his car to minimize the distance Toru will have to walk from the café, to the car, and then to the arena entrance? Draw a diagram.



Example 3 **Graph $\triangle ABC$ and its image in the given line.**

5. $y = -2$ **6.** $x = 3$



Examples 4–5 **Graph each figure and its image under the given reflection.**

7. $\triangle XYZ$ with vertices $X(0, 4)$, $Y(-3, 4)$, and $Z(-4, -1)$ in the y-axis

8. $\square QRST$ with vertices $Q(-1, 4)$, $R(4, 4)$, $S(3, 1)$, and $T(-2, 1)$ in the x-axis

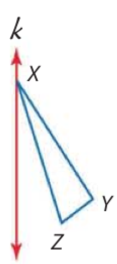
9. quadrilateral $JKLM$ with vertices $J(-3, 1)$, $K(-1, 3)$, $L(1, 3)$, and $M(-3, -1)$ in the line $y = x$

Practice and Problem Solving

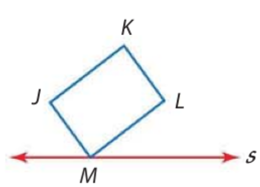
Extra Practice is on page R9.

Example 1 **CCSS TOOLS** **Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.**

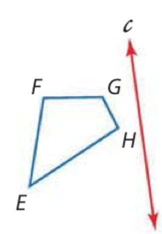
10.



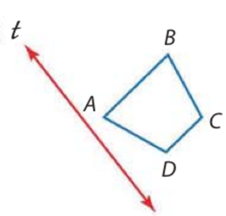
11.



12.



13.



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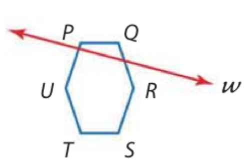
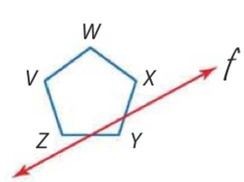
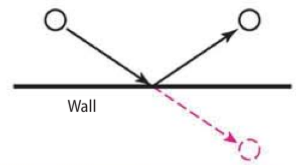


15.



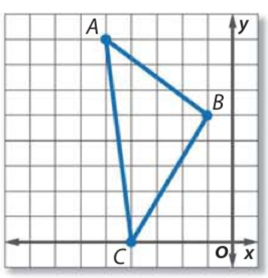
Example 2 **SPORTS** **When a ball is rolled or struck without spin against a wall, it bounces off the wall and travels in a ray that is the reflected image of the path of the ball if it had gone straight through the wall. Use this information in Exercises 16 and 17.**

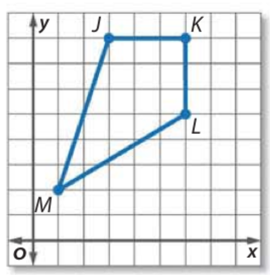


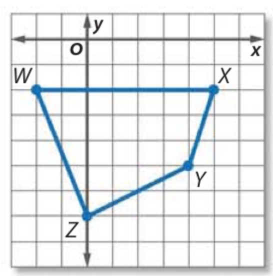
16. **BILLIARDS** Tadeo is playing billiards. He wants to strike the eight ball with the cue ball so that the eight ball bounces off the rail and rolls into the indicated pocket. If the eight ball moves with no spin, draw a diagram showing the exact point *P* along the right rail where the eight ball should hit after being struck by the cue ball.

17. **INDOOR SOCCER** Abby is playing indoor soccer, and she wants to hit the ball to point *C*, but must avoid an opposing player at point *B*. She decides to hit the ball at point *A* so that it bounces off the side wall. Draw a diagram that shows the exact point along the top wall for which Abby should aim.

Example 3 **Graph each figure and its image in the given line.**







18. $\triangle ABC$; $y = 3$

19. $\triangle ABC$; $x = -1$

20. *JKLM*; $x = 1$

21. *JKLM*; $y = 4$

22. *WXYZ*; $y = -4$

23. *WXYZ*; $x = -2$

Examples 4–5 **CCSS STRUCTURE** **Graph each figure and its image under the given reflection.**

24. rectangle *ABCD* with vertices $A(-5, 2)$, $B(1, 2)$, $C(1, -1)$, and $D(-5, -1)$ in the line $y = -2$

25. square *JKLM* with vertices $J(-4, 6)$, $K(0, 6)$, $L(0, 2)$, and $M(-4, 2)$ in the *y*-axis

26. $\triangle FGH$ with vertices $F(-3, 2)$, $G(-4, -1)$, and $H(-6, -1)$ in the line $y = x$

27. ▱*WXYZ* with vertices $W(2, 3)$, $X(7, 3)$, $Y(6, -1)$, and $Z(1, -1)$ in the *x*-axis

28. trapezoid *PQRS* with vertices $P(-1, 4)$, $Q(2, 4)$, $R(1, -1)$, and $S(-1, -1)$ in the *y*-axis

29. $\triangle STU$ with vertices $S(-3, -2)$, $T(-2, 3)$, and $U(2, 2)$ in the line $y = x$

Each figure shows a preimage and its reflected image in some line. Copy each figure and draw the line of reflection.

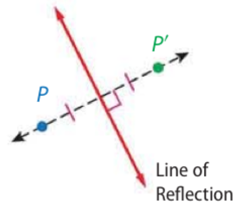
30.

31.

32.

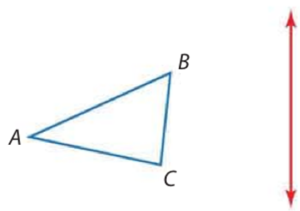
CONSTRUCTION To construct the reflection of a figure in a line using only a compass and a straightedge, you can use:

- the construction of a line perpendicular to a given line through a point not on the line (p. 55), and
- the construction of a segment congruent to a given segment (p. 17).

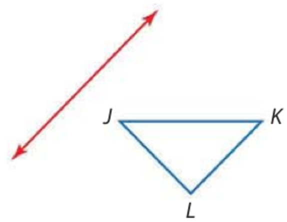


CCSS TOOLS Copy each figure and the given line of reflection. Then construct the reflected image.

33.



34.



35 **PHOTOGRAPHY** Refer to the photo at the right.

a. What object separates the zebras and their reflections?

b. What geometric term can be used to describe this object?

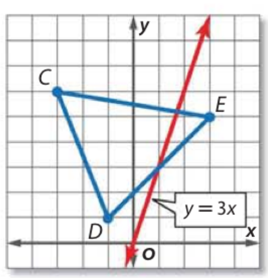
ALGEBRA Graph the line $y = 2x - 3$ and its reflected image in the given line. What is the equation of the reflected image?

36. x-axis

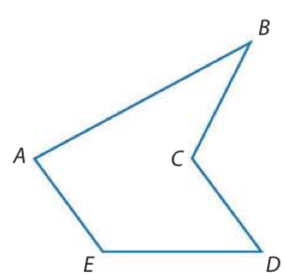
37. y-axis

38. $y = x$

39. Reflect $\triangle CDE$ shown below in the line $y = 3x$.



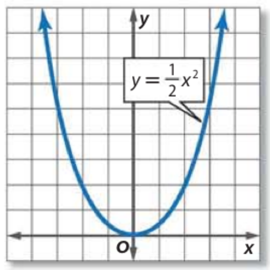
40. Relocate vertex C so that $ABCDE$ is convex, and all sides remain the same length.



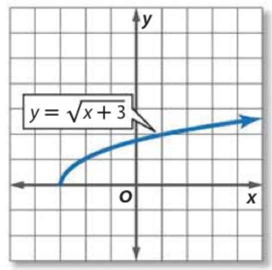
Digital Vision/Getty Images

ALGEBRA **Graph the reflection of each function in the given line. Then write the equation of the reflected image.**

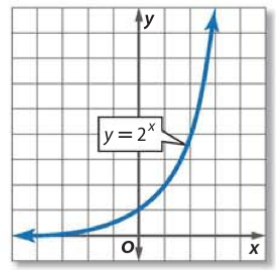
41. x-axis



42. y-axis



43. x-axis



44. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a reflection in the origin.

a. Geometric Draw $\triangle ABC$ in the coordinate plane so that each vertex is a whole-number ordered pair.

b. Graphical Locate each reflected point A', B', and C' so that the reflected point, the original point, and the origin are collinear, and both the original point and the reflected point are equidistant from the origin.

c. Tabular Copy and complete the table below.

	$\triangle ABC$		$\triangle A'B'C'$	
Coordinates	A		A'	
	B		B'	
	C		C'	

d. Verbal Make a conjecture about the relationship between corresponding vertices of a figure reflected in the origin.

H.O.T. Problems Use Higher-Order Thinking Skills

45. **ERROR ANALYSIS** Jamil and Ashley are finding the coordinates of the image of (2, 3) after a reflection in the x-axis. Is either of them correct? Explain.

Jamil	Ashley
$C'(2, -3)$	$C'(-2, 3)$

46. **WRITING IN MATH** Describe how to reflect a figure not on the coordinate plane across a line.

47. **CHALLENGE** A point in the second quadrant with coordinates $(-a, b)$ is reflected in the x-axis. If the reflected point is then reflected in the line $y = -x$, what are the final coordinates of the image?

48. **OPEN ENDED** Draw a polygon on the coordinate plane that when reflected in the x-axis looks exactly like the original figure.

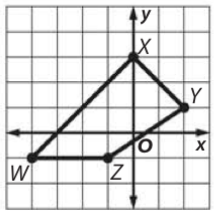
49. **CHALLENGE** When $A(4, 3)$ is reflected in a line, its image is $A'(-1, 0)$. Find the equation of the line of reflection. Explain your reasoning.

50. **CCSS PRECISION** The image of a point reflected in a line is *always, sometimes,* or *never* located on the other side of the line of reflection.

51. **WRITING IN MATH** Suppose points P, Q, and R are collinear, with point Q between points P and R. Describe a plan for a proof that the reflection of points P, Q, and R in a line preserves collinearity and betweenness of points.

Standardized Test Practice

52. SHORT RESPONSE If quadrilateral *WXYZ* is reflected across the *y*-axis to become quadrilateral *W′X′Y′Z′*, what are the coordinates of *X′*?

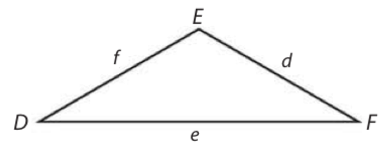


53. ALGEBRA If the arithmetic mean of $6x$, $3x$, and 27 is 18, then what is the value of x?

A 2   **C** 5

B 3   **D** 6

54. In $\triangle DEF$, $m\angle E = 108$, $m\angle F = 26$, and $f = 20$. Find d to the nearest whole number.



F 26   **G** 33   **H** 60   **J** 65

55. SAT/ACT In a coordinate plane, points *A* and *B* have coordinates (−2, 4) and (3, 3), respectively. What is the value of *AB*?

A $\sqrt{50}$ **D** (1, −1)

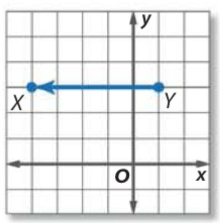
B (1, 7)   **E** $\sqrt{26}$

C (5, −1)

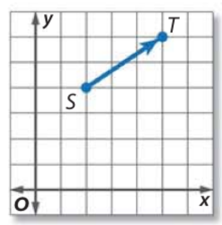
Spiral Review

Write the component form of each vector. (Lesson 8-7)

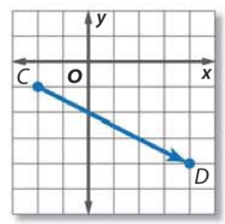
56.



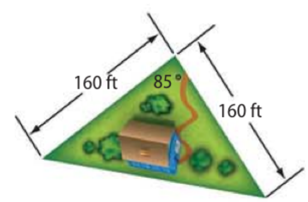
57.



58.

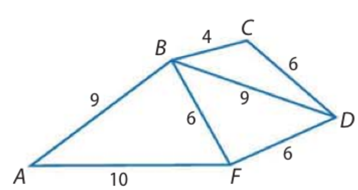


59. REAL ESTATE A house is built on a triangular plot of land. Two sides of the plot are 160 feet long, and they meet at an angle of 85°. If a fence is to be placed along the perimeter of the property, how much fencing material is needed? (Lesson 8-6)



60. COORDINATE GEOMETRY In $\triangle LMN$, $\overline{PR}$ divides $\overline{NL}$ and $\overline{MN}$ proportionally. If the vertices are $N(8, 20)$, $P(11, 16)$, and $R(3, 8)$ and $\frac{LP}{PN} = \frac{2}{1}$, find the coordinates of *L* and *M*. (Lesson 7-4)

Use the figure at the right to write an inequality relating the given pair of angle or segment measures. (Lesson 5-6)



61. *AB*, *FD*

62. $m\angle BDC$, $m\angle FDB$

63. $m\angle FBA$, $m\angle DBF$

Skills Review

Find the magnitude and direction of each vector.

64. $\overrightarrow{RS}$: $R(-3, 3)$ and $S(-9, 9)$

65. $\overrightarrow{JK}$: $J(8, 1)$ and $K(2, 5)$

66. $\overrightarrow{FG}$: $F(-4, 0)$ and $G(-6, -4)$

67. $\overrightarrow{AB}$: $A(-1, 10)$ and $B(1, -12)$

LESSON 9-2 Translations

Then

- You found the magnitude and direction of vectors.

Now

1. Draw translations.
2. Draw translations in the coordinate plane.

Why?

- Stop-motion animation is a technique in which an object is moved by very small amounts between individually photographed frames. When the series of frames is played as a continuous sequence, the result is the illusion of movement.

NewVocabulary
translation vector

Common Core State Standards

Content Standards

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematical Practices

5 Use appropriate tools strategically.

4 Model with mathematics.

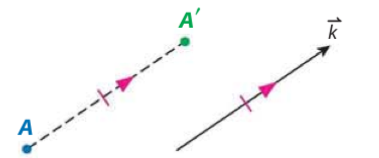
1 Draw Translations

In Lesson 4-7, you learned that a translation or *slide* is a transformation that moves all points of a figure the same distance in the same direction. Since vectors can be used to describe both distance and direction, vectors can be used to define translations.

KeyConcept Translation

A translation is a function that maps each point to its image along a vector, called the **translation vector**, such that

- each segment joining a point and its image has the same length as the vector, and
- this segment is also parallel to the vector.



Point *A′* is a translation of point *A* along translation vector $\vec{k}$.

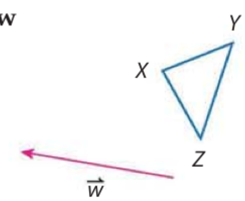
Example 1 Draw a Translation

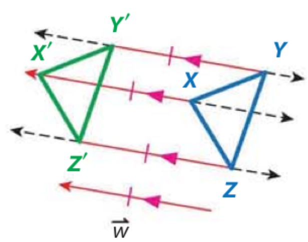
Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

Step 1 Draw a line through each vertex parallel to vector $\vec{w}$.

Step 2 Measure the length of vector $\vec{w}$. Locate point *X′* by marking off this distance along the line through vertex *X*, starting at *X* and in the same direction as the vector.

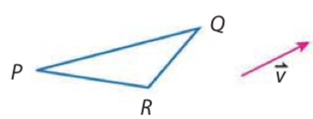
Step 3 Repeat Step 2 to locate points *Y′* and *Z′*. Then connect vertices *X′*, *Y′*, and *Z′* to form the translated image.



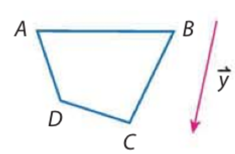


GuidedPractice

1A.

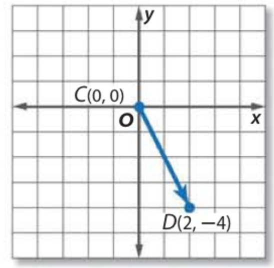


1B.



2 Draw Translations in the Coordinate Plane

Recall that a vector in the coordinate plane can be written as $\langle a, b \rangle$, where a represents the horizontal change and b is the vertical change from the vector's tip to its tail. $\overline{CD}$ is represented by the ordered pair $\langle 2, -4 \rangle$.



Written in this form, called the component form, a vector can be used to translate a figure in the coordinate plane.

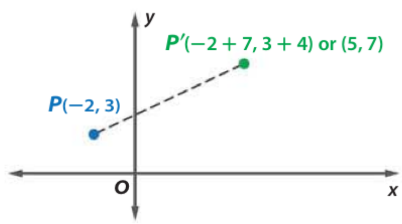
> **ReadingMath**
>
> **Horizontal and Vertical Translations** When the translation vector is of the form $\langle a, 0 \rangle$, the translation is horizontal only. When the translation vector is of the form $\langle 0, b \rangle$, the translation is vertical only.

KeyConcept Translation in the Coordinate Plane

Words To translate a point along vector $\langle a, b \rangle$, add a to the x-coordinate and b to the y-coordinate.

Symbols $(x, y) \rightarrow (x + a, y + b)$

Example The image of $P(-2, 3)$ translated along vector $\langle 7, 4 \rangle$ is $P'(5, 7)$.



A translation is another type of congruence transformation or isometry.

Example 2 Translations in the Coordinate Plane

Graph each figure and its image along the given vector.

a. **$\triangle EFG$ with vertices $E(-7, -1)$, $F(-4, -4)$, and $G(-3, -1)$; $\langle 2, 5 \rangle$**

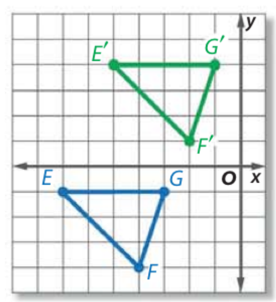
The vector indicates a translation 2 units right and 5 units up.

$(x, y) \rightarrow (x + 2, y + 5)$

$E(-7, -1) \rightarrow E'(-5, 4)$

$F(-4, -4) \rightarrow F'(-2, 1)$

$G(-3, -1) \rightarrow G'(-1, 4)$



b. **square $JKLM$ with vertices $J(3, 4)$, $K(5, 2)$, $L(7, 4)$, and $M(5, 6)$; $\langle -3, -4 \rangle$**

The vector indicates a translation 3 units left and 4 units down.

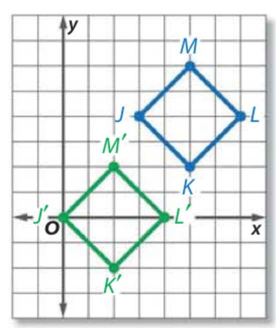
$(x, y) \rightarrow (x + (-3), y + (-4))$

$J(3, 4) \rightarrow J'(0, 0)$

$K(5, 2) \rightarrow K'(2, -2)$

$L(7, 4) \rightarrow L'(4, 0)$

$M(5, 6) \rightarrow M'(2, 2)$



GuidedPractice

2A. $\triangle ABC$ with vertices $A(2, 6)$, $B(1, 1)$, and $C(7, 5)$; $\langle -4, -1 \rangle$

2B. quadrilateral $QRST$ with vertices $Q(-8, -2)$, $R(-9, -5)$, $S(-4, -7)$, and $T(-4, -2)$; $\langle 7, 1 \rangle$

Science & Society Picture Library/Getty Images

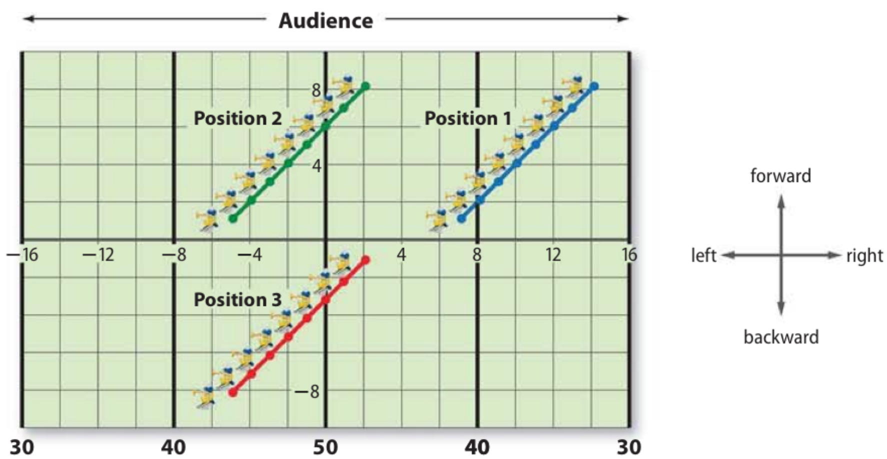
> **Math HistoryLink**
>
> **Felix Klein** (1849–1925) Klein's definition of geometry as the study of the properties of a space that remain invariant under a group of transformations allowed for the inclusion of both Euclidean and non-Euclidean geometry.

Real-WorldLink

Marching bands often make use of a series of formations that can include geometric shapes. Usually, each band member has an assigned position in each formation. *Floating* is the movement of a group of members together without changing the shape or size of their formation.

Real-World Example 3 Describing Translations

MARCHING BAND **In one part of a marching band's performance, a line of trumpet players starts at position 1, marches to position 2, and then to position 3. Each unit on the graph represents one step.**



a. Describe the translation of the trumpet line from position 1 to position 2 in function notation and in words.

One point on the line in position 1 is (14, 8). In position 2, this point moves to (2, 8). Use the translations function $(x, y) \rightarrow (x + a, y + b)$ to write and solve equations to find a and b.

$(14 + a, 8 + b)$ or (2, 8)

$14 + a = 2$ $\quad\quad$ $8 + b = 8$

$a = -12$ $\quad\quad$ $b = 0$

function notation: $(x, y) \rightarrow (x + (-12), y + 0)$

So, the trumpet line is translated 12 steps *left* but no steps forward or backward from position 1 to position 2.

b. Describe the translation of the line from position 1 to position 3 using a translation vector.

$(14 + a, 8 + b)$ or $(2, -1)$

$14 + a = 2$ $\quad\quad$ $8 + b = -1$

$a = -12$ $\quad\quad$ $b = -9$

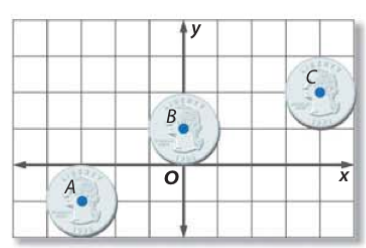
translation vector: $\langle -12, -9 \rangle$

GuidedPractice

3. **ANIMATION** A coin is filmed using stop-motion animation so that it appears to move.

A. Describe the translation from A to B in function notation and in words.

B. Describe the translation from A to C using a translation vector.

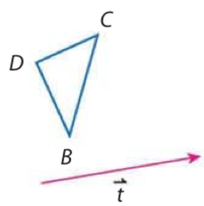


Check Your Understanding

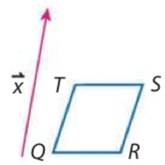
= Step-by-Step Solutions begin on page R14.

Example 1 **Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.**

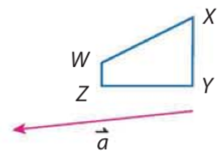
1.



2.



3.



Example 2 **Graph each figure and its image along the given vector.**

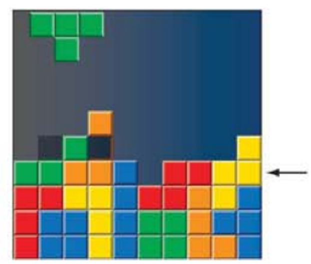
4. trapezoid $JKLM$ with vertices $J(2, 4)$, $K(1, 1)$, $L(5, 1)$ and $M(4, 4)$; $\langle 7, 1\rangle$

5. $\triangle DFG$ with vertices $D(-8, 8)$, $F(-10, 4)$, and $G(-7, 6)$; $\langle 5, -2\rangle$

6. parallelogram $WXYZ$ with vertices $W(-6, -5)$, $X(-2, -5)$, $Y(-1, -8)$, and $Z(-5, -8)$; $\langle -1, 4\rangle$

Example 3

7. **VIDEO GAMES** The object of the video game shown is to manipulate the colored tiles left or right as they fall from the top of the screen to completely fill each row without leaving empty spaces. If the starting position of the tile piece at the top of the screen is (x, y), use function notation to describe the translation that will fill the indicated row.

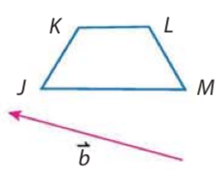


Practice and Problem Solving

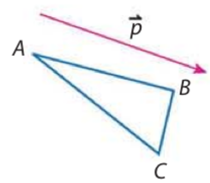
Extra Practice is on page R9.

Example 1 CCSS **TOOLS** **Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.**

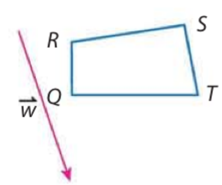
8.



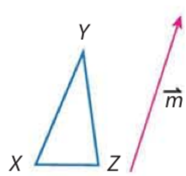
9.



10.



11.



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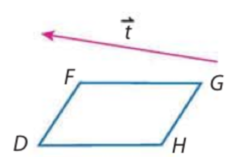

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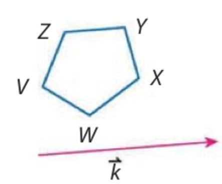


Example 2 **Graph each figure and its image along the given vector.**

14. $\triangle ABC$ with vertices $A(1, 6)$, $B(3, 2)$, and $C(4, 7)$; $\langle 4, -1\rangle$

15. $\triangle MNP$ with vertices $M(4, -5)$, $N(5, -8)$, and $P(8, -6)$; $\langle -2, 5\rangle$

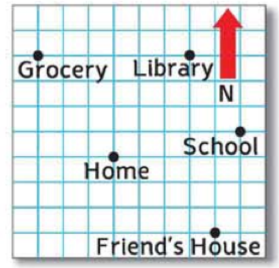
16. rectangle $QRST$ with vertices $Q(-8, 4)$, $R(-8, 2)$, $S(-3, 2)$, and $T(-3, 4)$; $\langle 2, 3\rangle$

17. quadrilateral $FGHJ$ with vertices $F(-4, -2)$, $G(-1, -1)$, $H(0, -4)$, and $J(-3, -6)$; $\langle -5, -2\rangle$

18. $\square WXYZ$ with vertices $W(-3, -1)$, $X(1, -1)$, $Y(2, -4)$, and $Z(-2, -4)$; $\langle -3, 4\rangle$

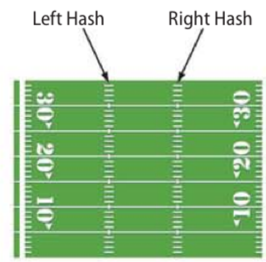
19. trapezoid $JKLM$ with vertices $J(-4, -2)$, $K(-1, -2)$, $L(0, -5)$, and $M(-5, -5)$; $\langle 6, 5\rangle$

Example 3 20. **MODELING** Brittany's neighborhood is shown on the grid at the right.



a. If she leaves home and travels 4 blocks north and 3 blocks east, what is her new location?

b. Use words to describe two possible translations that will take Brittany home from school.



21 **FOOTBALL** A wide receiver starts from his 15-yard line on the right hash mark and runs a route that takes him 12 yards to the left and down field for a gain of 17 yards. Write a translation vector to describe the receiver's route.

22. **CHESS** Each chess piece has a path that it can follow to move. The rook, which begins in square a8, can only move vertically or horizontally. The knight, which begins in square b8, can move two squares horizontally and then one square vertically, or two squares vertically and one square horizontally. The bishop, which begins in square f8, can only move diagonally.

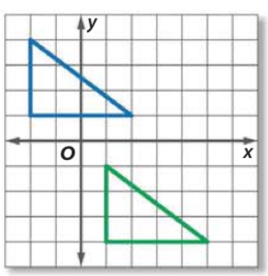
a. The knight moves 2 squares vertically and 1 square horizontally on its first move, then two squares horizontally and 1 square vertically on its second move. What are the possible locations for the knight after two moves?

b. After two moves, the rook is in square d3. Describe a possible translation to describe the two moves.

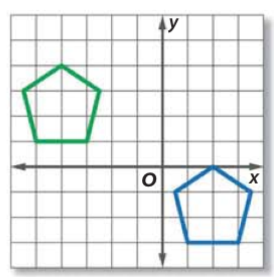
c. Describe a translation that can take the bishop to square a1. What is the minimum number of moves that can be used to accomplish this translation?

Write each translation vector.

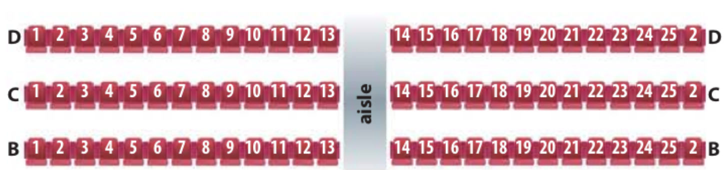
23.



24.

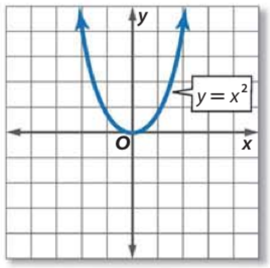


25. **CONCERTS** Dexter's family buys tickets every year for a concert. Last year they were in seats C3, C4, C5, and C6. This year, they will be in seats D16, D17, D18, and D19. Write a translation in words and using vector notation that can be used to describe the change in their seating.

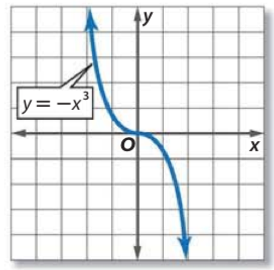


SENSE-MAKING Graph the translation of each function along the given vector. Then write the equation of the translated image.

26. $\langle 4, 1 \rangle$



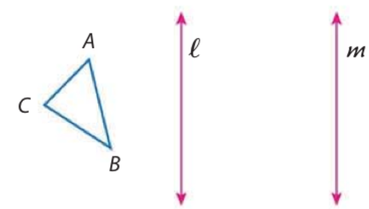
27. $\langle -2, 0 \rangle$



28. ROLLER COASTERS The length of the roller coaster track from the top of a hill to the bottom of the hill is 125 feet at a 53° angle with the vertical. If the position at the top of the hill is (x, y), use function notation to describe the translation to the bottom of the hill. Round to the nearest foot.

29. MULTIPLE REPRESENTATIONS In this problem, you will investigate reflections over a pair of parallel lines.

a. Geometric On patty paper, draw $\triangle ABC$ and a pair of vertical lines ℓ and m. Reflect $\triangle ABC$ in line ℓ by folding the patty paper. Then reflect $\triangle A'B'C'$, in line m. Label the final image $\triangle A''B''C''$.



b. Geometric Repeat the process in part **a** for $\triangle DEF$ reflected in vertical lines n and p and $\triangle JKL$ reflected in vertical lines q and r.

c. Tabular Copy and complete the table below.

Distance Between Corresponding Points (cm)		Distance Between Vertical Lines (cm)	
A and A'', B and B'', C and C''		ℓ and m	
D and D'', E and E'', F and F''		n and p	
J and J'', K and K'', L and L''		q and r	

d. Verbal Describe the result of two reflections in two vertical lines using one transformation.

H.O.T. Problems Use Higher-Order Thinking Skills

30. REASONING Determine a rule to find the final image of a point that is translated along $\langle x + a, y + b \rangle$ and then $\langle x + c, y + d \rangle$.

31. CHALLENGE A line $y = mx + b$ is translated using the vector $\langle a, b \rangle$. Write the equation of the translated line. What is the value of the y-intercept?

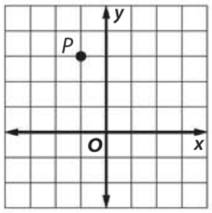
32. OPEN ENDED Draw a figure on the coordinate plane so that the figure has the same orientation after it is reflected in the line $y = 1$. Explain what must be true in order for this to occur.

33. WRITING IN MATH Compare and contrast function notation and vector notation for translations.

34. WRITING IN MATH Recall from Lesson 9-1 that an invariant point maps onto itself. Can invariant points occur with translations? Explain why or why not.

Standardized Test Practice

35. Identify the location of point *P* under translation $(x + 3, y + 1)$.



A (0, 6) **C** (2, −4)
B (0, 3) **D** (2, 4)

36. SHORT RESPONSE Which vector best describes the translation of $A(3, -5)$ to $A'(-2, -8)$?

37. ALGEBRA Over the next four days, Amanda plans to drive 160 miles, 235 miles, 185 miles, and 220 miles. If her car gets an average of 32 miles per gallon of gas, how many gallons of gas should she expect to use in all?

F 25 **G** 30 **H** 35 **J** 40

38. SAT/ACT A bag contains 5 red marbles, 2 blue marbles, 4 white marbles, and 1 yellow marble. If two marbles are chosen in a row, without replacement, what is the probability of getting 2 white marbles?

A $\frac{1}{66}$ **C** $\frac{1}{9}$ **E** $\frac{2}{5}$
B $\frac{1}{11}$ **D** $\frac{5}{33}$

Spiral Review

Graph each figure and its image under the given reflection. (Lesson 9-1)

39. $\overline{DJ}$ with endpoints $D(4, 4)$, $J(-3, 2)$ in the *y*-axis

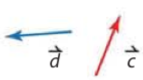
40. $\triangle XYZ$ with vertices $X(0, 0)$, $Y(3, 0)$, and $Z(0, 3)$ in the *x*-axis

41. $\triangle ABC$ with vertices $A(-3, -1)$, $B(0, 2)$, and $C(3, -2)$, in the line $y = x$

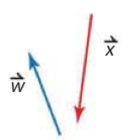
42. quadrilateral *JKLM* with vertices $J(-2, 2)$, $K(3, 1)$, $L(4, -1)$, and $M(-2, -2)$ in the origin

Copy the vectors to find each sum or difference. (Lesson 8-7)

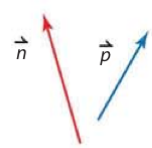
43. $\vec{c} + \vec{d}$



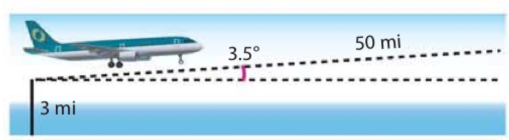
44. $\vec{w} + \vec{x}$



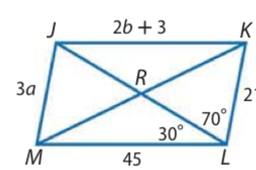
45. $\vec{n} - \vec{p}$



46. NAVIGATION An airplane is three miles above sea level when it begins to climb at a 3.5° angle. If this angle is constant, how far above sea level is the airplane after flying 50 miles? (Lesson 8-4)



Use ▱*JKLM* to find each measure. (Lesson 6-2)

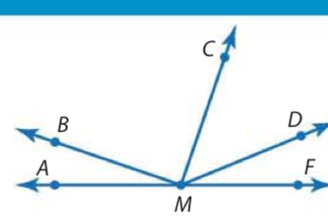


47. $m\angle MJK$ **48.** $m\angle JML$

49. $m\angle JKL$ **50.** $m\angle KJL$

Skills Review

Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



51. $\angle AMC$ **52.** $\angle FMD$

53. $\angle BMD$ **54.** $\angle CMB$

EXPLORE 9-3

Geometry Lab
Rotations

In Chapter 4, you learned that a rotation is a type of transformation that moves a figure about a fixed point, or center of rotation, through a specific angle and in a specific direction. In this activity you will use tracing paper to explore the properties of rotations.

CCSS Common Core State Standards
Content Standards
G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Mathematical Practices 5

Activity Explore Rotations by Using Patty Paper

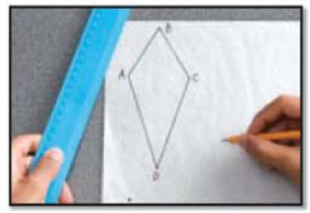
Step 1 On a piece of tracing paper, draw quadrilateral *ABCD* and a point *P*.

Step 2 On another piece of tracing paper, trace quadrilateral *ABCD* and point *P*. Label the new quadrilateral *A′B′C′D′* and the new point *P*.

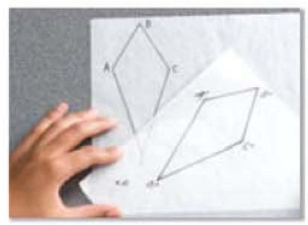
Step 3 Position the tracing paper so that both points *P* coincide. Rotate the paper so that *ABCD* and *A′B′C′D′* do not overlap. Tape the two pieces of tracing paper together.

Step 4 Measure the distance between *A*, *B*, *C*, and *D* to point *P*. Repeat for quadrilateral *A′B′C′D′*. Then copy and complete the table below.

Quadrilateral	Length			
ABCD	AP	BP	CP	DP
A′B′C′D′	A′P	B′P	C′P	D′P



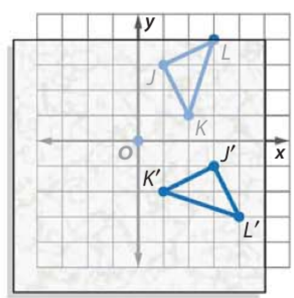
Step 1



Steps 2 and 3

Exercises

1. Graph △*JKL* with vertices *J*(1, 3), *K*(2, 1), and *L*(3, 4) on a coordinate plane, and then trace on tracing paper.



   a. Use a protractor to rotate each vertex 90° clockwise about the origin as shown in the figure at the right. What are the vertices of the rotated image?

   b. Rotate △*JKL* 180° about the origin. What are the vertices of the rotated image?

   c. Use the Distance Formula to find the distance from points *J*, *K*, and *L* to the origin. Repeat for *J′K′L′* and *J″K″L″*.

2. **WRITING IN MATH** If you rotate point (4, 2) 90° and 180° about the origin, how do the *x*- and *y*-coordinates change?

3. **MAKE A PREDICTION** What are the new coordinates of a point (*x*, *y*) that is rotated 270°?

4. **MAKE A CONJECTURE** Make a conjecture about the distances from the center of rotation *P* to each corresponding vertex of *ABCD* and *A′B′C′D′*.

LESSON 9-3

Rotations

:: Then	:: Now	:: Why?
You identified rotations and verified them as congruence transformations.	1 Draw rotations. 2 Draw rotations in the coordinate plane.	Modern windmill technology may be an important alternative to fossil fuels. Windmills convert the wind's energy into electricity through the rotation of turbine blades.

NewVocabulary
center of rotation
angle of rotation

Common Core State Standards

Content Standards

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematical Practices

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

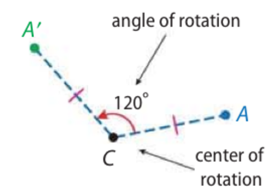
1 Draw Rotations

In Lesson 4-7, you learned that a rotation or *turn* moves every point of a preimage through a specified angle and direction about a fixed point.

KeyConcept Rotation

A rotation about a fixed point, called the **center of rotation**, through an angle of $x°$ is a function that maps a point to its image such that

- if the point is the center of rotation, then the image and preimage are the same point, or
- if the point is not the center of rotation, then the image and preimage are the same distance from the center of rotation and the measure of the **angle of rotation** formed by the preimage, center of rotation, and image points is x.

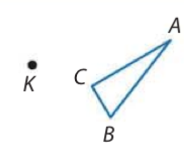


A' is the image of A after a 120° rotation about point C.

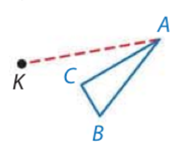
The direction of a rotation can be either clockwise or counterclockwise. Assume that all rotations are counterclockwise unless stated otherwise.

Example 1 Draw a Rotation

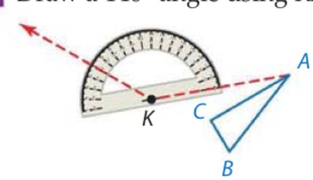
Copy $\triangle ABC$ and point K. Then use a protractor and ruler to draw a 140° rotation of $\triangle ABC$ about point K.



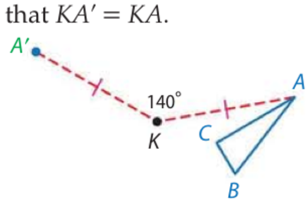
Step 1 Draw a segment from A to K.



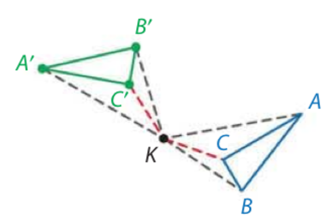
Step 2 Draw a 140° angle using $\overline{KA}$.



Step 3 Use a ruler to draw A' such that $KA' = KA$.



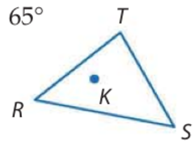
Step 4 Repeat Steps 1–3 for vertices B and C and draw $\triangle A'B'C'$.



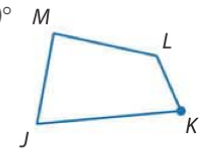
GuidedPractice

Copy each figure and point *K*. Then use a protractor and ruler to draw a rotation of the figure the given number of degrees about *K*.

1A. 65°



1B. 170°



> **Study**Tip
>
> **Clockwise Rotation** Clockwise rotation can be designated by a negative angle measure. For example a rotation of −90° about the origin is a rotation 90° clockwise about the origin.

2 Draw Rotations in the Coordinate Plane

When a point is rotated 90°, 180°, or 270° counterclockwise about the origin, you can use the following rules.

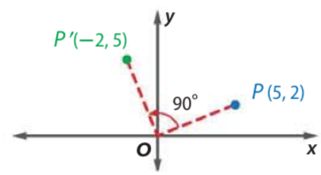
KeyConcept Rotations in the Coordinate Plane

90° Rotation

To rotate a point 90° counterclockwise about the origin, multiply the *y*-coordinate by −1 and then interchange the *x*- and *y*-coordinates.

Symbols $(x, y) \rightarrow (-y, x)$

Example

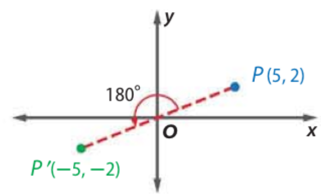


180° Rotation

To rotate a point 180° counterclockwise about the origin, multiply the *x*- and *y*-coordinates by −1.

Symbols $(x, y) \rightarrow (-x, -y)$

Example

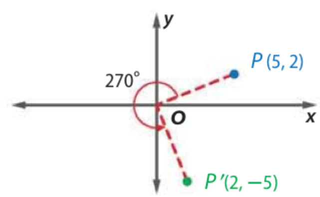


270° Rotation

To rotate a point 270° counterclockwise about the origin, multiply the *x*-coordinate by −1 and then interchange the *x*- and *y*-coordinates.

Symbols $(x, y) \rightarrow (y, -x)$

Example



> **Study**Tip
>
> **360° Rotation** A rotation of 360° about a point returns a figure to its original position. That is, the image under a 360° rotation is equal to the preimage.

Example 2 Rotations in the Coordinate Plane

Triangle *PQR* has vertices *P*(1, 1), *Q*(4, 5), and *R*(5, 1). Graph △*PQR* and its image after a rotation 90° about the origin.

Multiply the *y*-coordinate of each vertex by −1 and interchange.

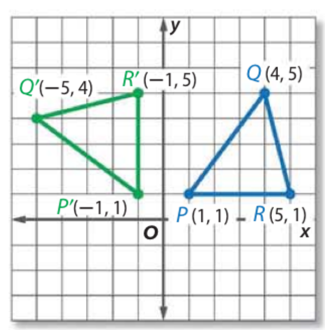
$(x, y) \rightarrow (-y, x)$

$P(1, 1) \rightarrow P'(-1, 1)$

$Q(4, 5) \rightarrow Q'(-5, 4)$

$R(5, 1) \rightarrow R'(-1, 5)$

Graph △*PQR* and its image △*P′Q′R′*.

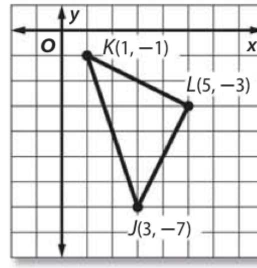


GuidedPractice

2. Parallelogram *FGHJ* has vertices *F*(2, 1), *G*(7, 1), *H*(6, −3), and *J*(1, −3). Graph *FGHJ* and its image after a rotation 180° about the origin.

Standardized Test Example 3 Rotations in the Coordinate Plane

Triangle *JKL* is shown at the right. What is the image of point *J* after a rotation 270° counterclockwise about the origin?



A (−3, −7)

B (−7, 3)

C (−7, −3)

D (7, −3)

Read the Test Item

You are given that △*JKL* has coordinates *J*(3, −7), *K*(1, −1), and *L*(5, −3) and are then asked to identify the coordinates of the image of point *J* after a 270° counterclockwise rotation about the origin.

Solve the Test Item

To find the coordinates of point *J* after a 270° counterclockwise rotation about the origin, multiply the *x*-coordinate by −1 and then interchange the *x*- and *y*-coordinates.

$(x, y) \rightarrow (y, -x)$ $(3, -7) \rightarrow (-7, -3)$

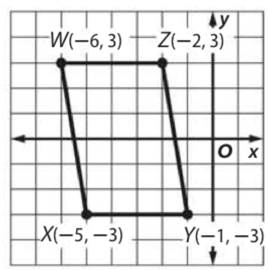
The answer is choice C.

StudyTip

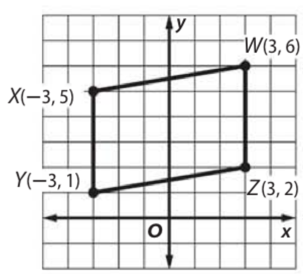
270° Rotation You can complete a 270° rotation by performing a 90° rotation and a 180° rotation in sequence.

GuidedPractice

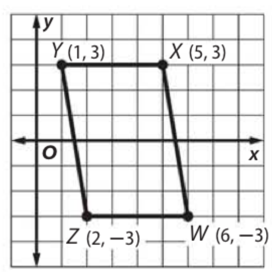
3. Parallelogram *WXYZ* is rotated 180° counterclockwise about the origin. Which of these graphs represents the resulting image?



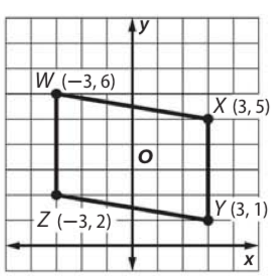
F



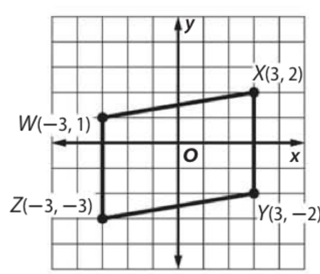
H



G



J



Test-TakingTip

CCSS Sense-Making

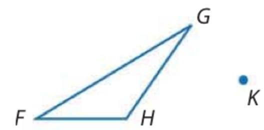
Instead of checking all four vertices of parallelogram *WXYZ* in each graph, check just one vertex, such as *X*.

Check Your Understanding

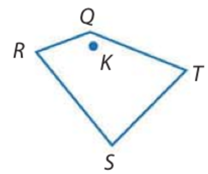
= Step-by-Step Solutions begin on page R14.

Example 1 **Copy each polygon and point *K*. Then use a protractor and ruler to draw the specified rotation of each figure about point *K*.**

1. 45°



2. 120°



Example 2 **3** Triangle *DFG* has vertices *D*(−2, 6), *F*(2, 8), and *G*(2, 3). Graph △*DFG* and its image after a rotation 180° about the origin.

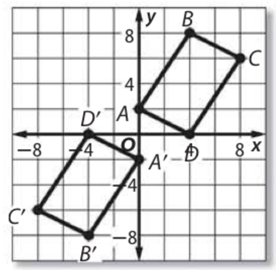
Example 3 **4. MULTIPLE CHOICE** For the transformation shown, what is the measure of the angle of rotation of *ABCD* about the origin?

A 90°

B 180°

C 270°

D 360°

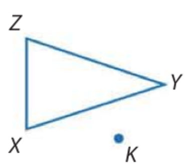


Practice and Problem Solving

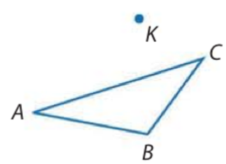
Extra Practice is on page R9.

Example 1 **CCSS TOOLS** **Copy each polygon and point *K*. Then use a protractor and ruler to draw the specified rotation of each figure about point *K*.**

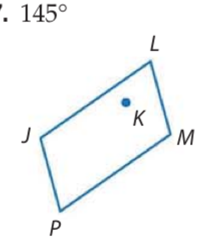
5. 90°



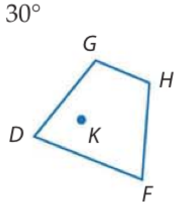
6. 15°



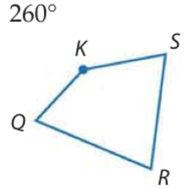
7. 145°



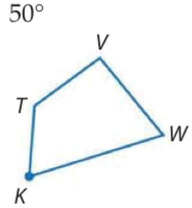
8. 30°



9. 260°

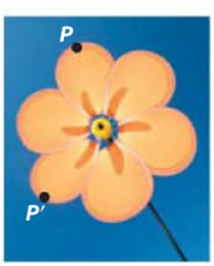


10. 50°



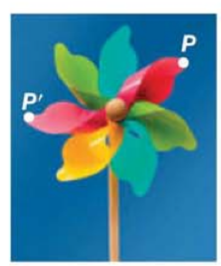
PINWHEELS **Find the angle of rotation to the nearest tenth of a degree that maps *P* onto *P*′. Explain your reasoning.**

11.



12.

13.



Examples 2–3 **Graph each figure and its image after the specified rotation about the origin.**

14. $\triangle JKL$ has vertices $J(2, 6)$, $K(5, 2)$, and $L(7, 5)$; 90°

15. rhombus $WXYZ$ has vertices $W(-3, 4)$, $X(0, 7)$, $Y(3, 4)$, and $Z(0, 1)$; 90°

16. $\triangle FGH$ has vertices $F(2, 4)$, $G(5, 6)$, and $H(7, 2)$; 180°

17. trapezoid $ABCD$ has vertices $A(-7, -2)$, $B(-6, -6)$, $C(-1, -1)$, and $D(-5, 0)$; 180°

18. $\triangle RST$ has vertices $R(-6, -1)$, $S(-4, -5)$, and $T(-2, -1)$; 270°

19. parallelogram $MPQV$ has vertices $M(-6, 3)$, $P(-2, 3)$, $Q(-3, -2)$, and $V(-7, -2)$; 270°

20. **WEATHER** A weathervane is used to indicate the direction of the wind. If the vane is pointing northeast and rotates 270°, what is the new wind direction?

21. **CCSS MODELING** The photograph of the Grande Roue, or Big Wheel, at the right appears blurred because of the camera's shutter speed—the length of time the camera's shutter was open. The diameter of the wheel is 60 meters.

a. Estimate the angle of rotation in the photo. (*Hint:* Use points A and A'.)

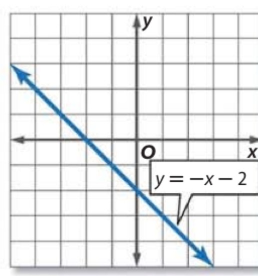
b. If the Ferris wheel makes one revolution per minute, use your estimate from part **a** to estimate the camera's shutter speed.

Each figure shows a preimage and its image after a rotation about point P. Copy each figure, locate point P, and find the angle of rotation.

22.

23.

ALGEBRA **Give the equation of the line $y = -x - 2$ after a rotation about the origin through the given angle. Then describe the relationship between the equations of the image and preimage.**



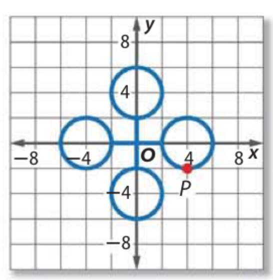
24. 90° **25.** 180°

26. 270° **27.** 360°

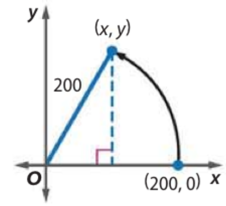
ALGEBRA **Rotate the line the specified number of degrees about the x- and y-intercepts and find the equation of the resulting image.**

28. $y = x - 5$; 90° **29.** $y = 2x + 4$; 180° **30.** $y = 3x - 2$; 270°

31 **RIDES** An amusement park ride consists of four circular cars. The ride rotates at a rate of 0.25 revolution per second. In addition, each car rotates 0.5 revolution per second. If Jane is positioned at point P when the ride begins, what coordinates describe her position after 31 seconds?



32. **BICYCLE RACING** Brandon and Nestor are participating in a bicycle race on a circular track with a radius of 200 feet.

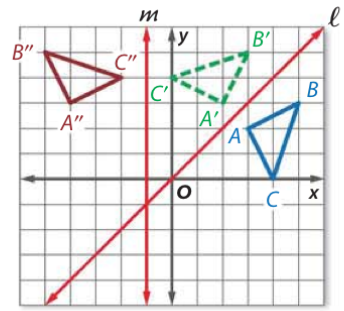


a. If the race starts at (200, 0) and both complete one rotation in 30 seconds, what are their coordinates after 5 seconds?

b. Suppose the length of race is 50 laps and Brandon continues the race at the same rate. If Nestor finishes in 26.2 minutes, who is the winner?

33. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate reflections over a pair of intersecting lines.

a. Geometric On a coordinate plane, draw a triangle and a pair of intersecting lines. Label the triangle ABC and the lines ℓ and m. Reflect $\triangle ABC$ in the line ℓ. Then reflect $\triangle A'B'C'$ in the line m. Label the final image $A''B''C''$.



b. Geometric Repeat the process in part a two more times in two different quadrants. Label the second triangle DEF and reflect it in intersecting lines n and p. Label the third triangle MNP and reflect it in intersecting lines q and r.

c. Tabular Measure the angle of rotation of each triangle about the point of intersection of the two lines. Copy and complete the table below.

Angle of Rotation Between Figures		Angle Between Intersecting Lines	
$\triangle ABC$ and $\triangle A''B''C''$		ℓ and m	
$\triangle DEF$ and $\triangle D''E''F''$		n and p	
$\triangle MNP$ and $\triangle M''N''P''$		q and r	

d. Verbal Make a conjecture about the angle of rotation of a figure about the intersection of two lines after the figure is reflected in both lines.

H.O.T. Problems Use Higher-Order Thinking Skills

34. **WRITING IN MATH** Are collinearity and betweenness of points maintained under rotation? Explain.

35. **CHALLENGE** Point C has coordinates $C(5, 5)$. The image of this point after a rotation of 100° about a certain point is $C'(-5, 7.5)$. Use construction to estimate the coordinates of the center of this rotation. Explain.

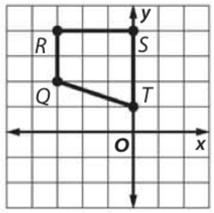
36. **OPEN ENDED** Draw a figure on the coordinate plane. Describe a nonzero rotation that maps the image onto the preimage with no change in orientation.

37. CCSS **ARGUMENTS** Is the reflection of a figure in the x-axis equivalent to the rotation of that same figure 180° about the origin? Explain.

38. **WRITING IN MATH** Do invariant points *sometimes*, *always*, or *never* occur in a rotation? Explain your reasoning.

Standardized Test Practice

39. What rotation of trapezoid *QRST* creates an image with point *R′* at (4, 3)?



A 270° counterclockwise about point *T*

B 185° counterclockwise about point *T*

C 180° clockwise about the origin

D 90° clockwise about the origin

40. SHORT RESPONSE △*XYZ* has vertices *X*(1, 7), *Y*(0, 2), and *Z*(−5, −2). What are the coordinates of *X′* after a rotation 270° counterclockwise about the origin?

41. ALGEBRA The population of the United States in July of 2007 was estimated to have surpassed 301,000,000. At the same time the world population was estimated to be over 6,602,000,000. What percent of the world population, to the nearest tenth, lived in the United States at this time?

F 3.1% **H** 4.2%

G 3.5% **J** 4.6%

42. SAT/ACT An 18-foot ladder is placed against the side of a house. The base of the ladder is positioned 8 feet from the house. How high up on the side of the house, to the nearest tenth of a foot, does the ladder reach?

A 10.0 ft **D** 22.5 ft

B 16.1 ft **E** 26.0 ft

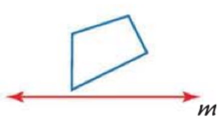
C 19.7 ft

Spiral Review

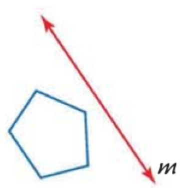
43. VOLCANOES A cloud of dense gas and dust from a volcano blows 40 miles west and then 30 miles north. Make a sketch to show the translation of the dust particles. Then find the distance of the shortest path that would take the particles to the same position. (Lesson 9-2)

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler. (Lesson 9-1)

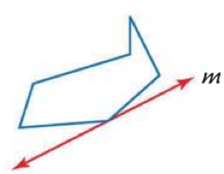
44.



45.



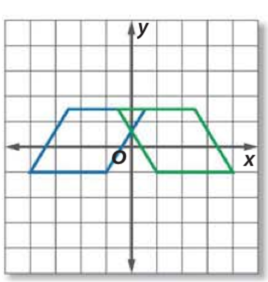
46.



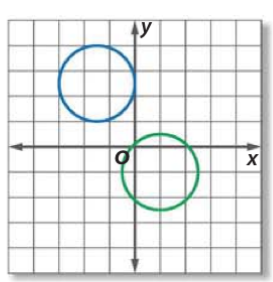
Skills Review

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

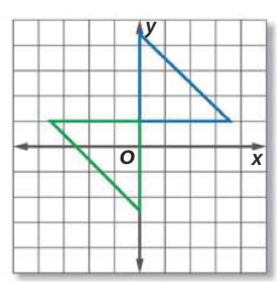
47.



48.



49.



EXTEND 9-3

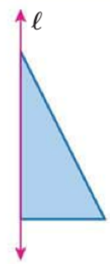
Geometry Lab
Solids of Revolution

A **solid of revolution** is a three-dimensional figure obtained by rotating a plane figure or curve about a line.

CCSS Common Core State Standards
Content Standards
G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Mathematical Practices 5

Activity 1

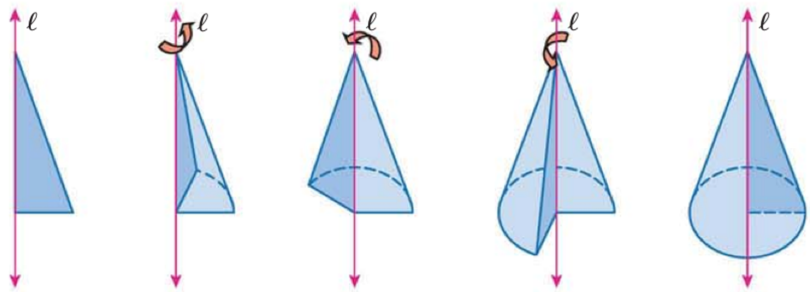
Identify and sketch the solid formed by rotating the right triangle shown about line ℓ.



Step 1 Copy the triangle onto card stock or heavy construction paper and cut it out.

Step 2 Use tape to attach the triangle to a dowel rod or straw.

Step 3 Rotate the end of the straw quickly between your hands and observe the result.

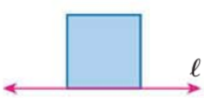


The blurred image you observe is that of a cone.

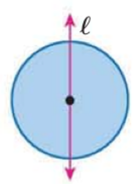
Model and Analyze

Identify and sketch the solid formed by rotating the two-dimensional shape about line ℓ.

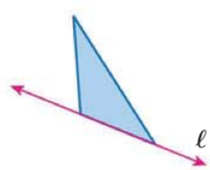
1.



2.



3.



4. Sketch and identify the solid formed by rotating the rectangle shown about the line containing

a. side $\overline{AB}$.

b. side $\overline{AD}$.

c. the midpoints of sides $\overline{AB}$ and $\overline{AD}$.

5. DESIGN Draw a two-dimensional figure that could be rotated to form the vase shown, including the line in which it should be rotated.

6. REASONING *True* or *false:* All solids can be formed by rotating a two-dimensional figure. Explain your reasoning.

Geometry Lab

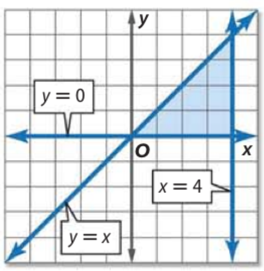
Solids of Revolution *Continued*

In calculus, you will be asked to find the volumes of solids generated by revolving a region on the coordinate plane about the x- or y-axis. An important first step in solving these problems is visualizing the solids formed.

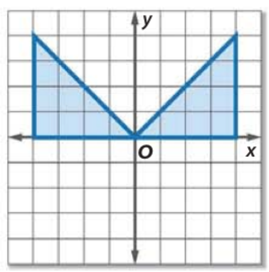
Activity 2

Sketch the solid that results when the region enclosed by $y = x$, $x = 4$, and $y = 0$ is revolved about the y-axis.

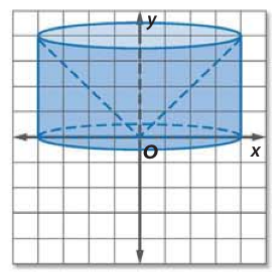
Step 1 Graph each equation to find the region to be rotated.



Step 2 Reflect the region about the y-axis.



Step 3 Connect the vertices of the right triangles using curved lines.



The solid is a cylinder with a cone cut out of its center.

Model and Analyze

Sketch the solid that results when the region enclosed by the given equations is revolved about the y-axis.

7. $y = -x + 4$
$x = 0$
$y = 0$

8. $y = x^2$
$y = 4$

9. $y = x^2$
$y = 2x$

Sketch the solid that results when the region enclosed by the given equations is revolved about the x-axis.

10. $y = -x + 4$
$x = 0$
$y = 0$

11. $y = x^2$
$y = 0$
$x = 2$

12. $y = x^2$
$y = 2x$

13. **OPEN ENDED** Graph a region in the first quadrant of the coordinate plane.

a. Sketch the graph of the region when revolved about the y-axis.

b. Sketch the graph of the region when revolved about the x-axis.

14. **CHALLENGE** Find equations that enclose a region such that when rotated about the x-axis, a solid is produced with a volume of 18π cubic units.

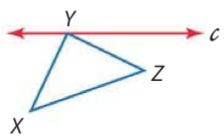
CHAPTER 9

Mid-Chapter Quiz

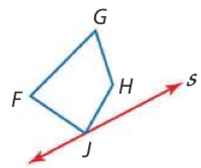
Lessons 9-1 through 9-3

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler. (Lesson 9-1)

1.



2.

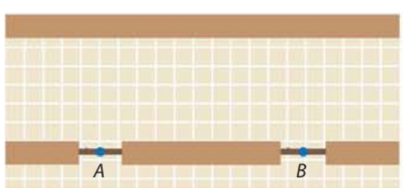


Graph each figure and its image after the specified reflection. (Lesson 9-1)

3. △*FGH* has vertices *F*(−4, 3), *G*(−2, 0), and *H*(−1, 4); in the *y*-axis

4. rhombus *QRST* has vertices *Q*(2, 1), *R*(4, 3), *S*(6, 1), and *T*(4, −1); in the *x*-axis

5. **CLUBS** The drama club is selling candy during the intermission of a school play. Locate point *P* along the wall to represent the candy table so that people coming from either door *A* or door *B* would walk the same distance to the table. (Lesson 9-1)



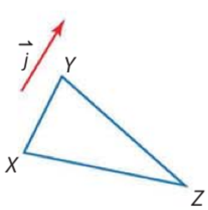
Graph each figure and its image after the specified translation. (Lesson 9-2)

6. △*ABC* with vertices *A*(0, 0), *B*(2, 1), *C*(1, −3); ⟨3, −1⟩

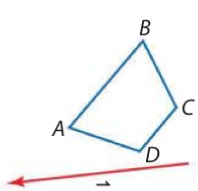
7. rectangle *JKLM* has vertices *J*(−4, 2), *K*(−4, −2), *L*(−1, −2), and *M*(−1, 2); ⟨5, −3⟩

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector. (Lesson 9-2)

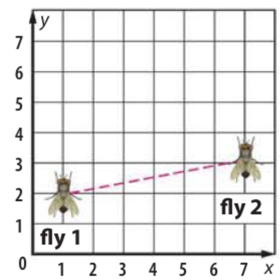
8.



9.

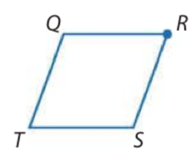


10. **COMICS** Alex is making a comic. He uses graph paper to make sure the dimensions of his drawings are accurate. If he draws a coordinate plane with two flies as shown below, what vector represents the movement from fly 1 to fly 2? (Lesson 9-2)

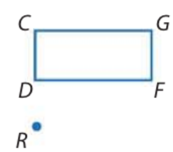


Copy each polygon and point *R*. Then use a protractor and ruler to draw the specified rotation of each figure about point *R*. (Lesson 9-3)

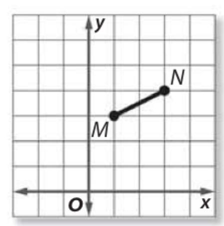
11. 45°



12. 60°



13. **MULTIPLE CHOICE** What is the image of point *M* after a rotation of 90° about the origin? (Lesson 9-3)



A (−3, 1)

B (−3, −1)

C (−1, −3)

D (3, 1)

Graph each figure and its image after the specified rotation. (Lesson 9-3)

14. △*RST* has vertices *R*(−3, 0), *S*(−1, −4), and *T*(0, −1); 90°

15. square *JKLM* has vertices *J*(−1, 2), *K*(−1, −2), *L*(3, −2), and *M*(3, 2); 180°

EXPLORE 9-4 Geometry Software Lab
Compositions of Transformations

In this lab, you will use Geometer's Sketchpad to explore the effects of performing multiple transformations on a figure.

CCSS Common Core State Standards

Content Standards

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematical Practices 5

Activity

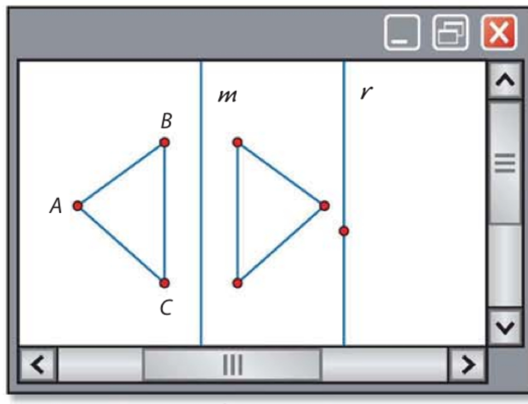
Reflect a figure in two vertical lines.

Step 1 Use the line segment tool to construct a triangle with one vertex pointing to the left so that you can easily see changes as you perform transformations. Label the triangle *ABC*.

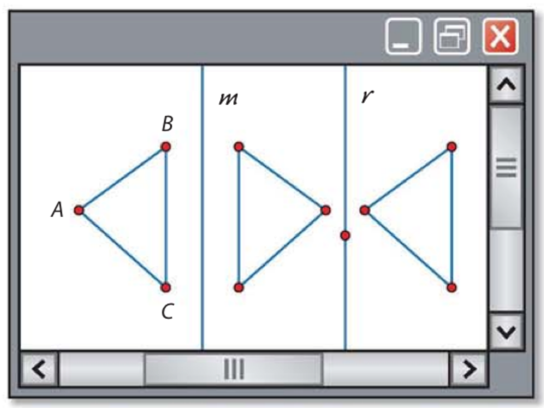
Step 2 Insert and label a line *m* to the right of △*ABC*. Insert a point so that the distance from the point to line *m* is greater than the width of △*ABC*. Draw the line parallel to line *m* through the point and label the new line *r*.

Step 3 Select line *m* and choose **Mark Mirror** from the **Transform** menu. Select all sides and vertices of △*ABC* and choose **Reflect** from the **Transform** menu.

Step 4 Repeat the process you used in Step 3 to reflect the new image in line *r*.



Steps 1–3



Step 4

Analyze the Results

1. How are the original figure and the final figure related?
2. What single transformation could be used to produce the final figure?
3. If you move line *m*, what happens? if you move line *r*?
4. **MAKE A CONJECTURE** If you reflected the figure in a third line, what single transformation do you think could be used to produce the final figure? Explain your reasoning.
5. Repeat the activity for a pair of perpendicular lines. What single transformation could be used to produce the same final figure?
6. **MAKE A CONJECTURE** If you reflected the figure from Exercise 5 in a third line perpendicular to the second line, what single transformation do you think could be used to produce the final figure? Explain your reasoning.

LESSON 9-4 Compositions of Transformations

Then

- You drew reflections, translations, and rotations.

Now

1. Draw glide reflections and other compositions of isometries in the coordinate plane.
2. Draw compositions of reflections in parallel and intersecting lines.

Why?

- The pattern of footprints left in the sand after a person walks along the edge of a beach illustrates the composition of two different transformations—translations and reflections.

NewVocabulary

composition of transformations
glide reflection

Common Core State Standards

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G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematical Practices

1 Make sense of problems and persevere in solving them.
4 Model with mathematics.

1 Glide Reflections

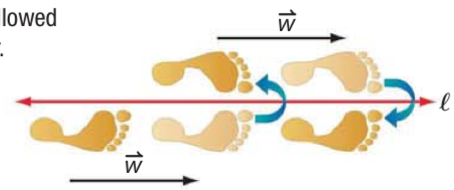
When a transformation is applied to a figure and then another transformation is applied to its image, the result is called a **composition of transformations**. A glide reflection is one type of composition of transformations.

KeyConcept Glide Reflection

A **glide reflection** is the composition of a translation followed by a reflection in a line parallel to the translation vector.

Example

The glide reflection shown is the composition of a translation along $\vec{w}$ followed by a reflection in line ℓ.



Example 1 Graph a Glide Reflection

Triangle *JKL* has vertices *J*(6, −1), *K*(10, −2), and *L*(5, −3). Graph △*JKL* and its image after a translation along ⟨0, 4⟩ and a reflection in the *y*-axis.

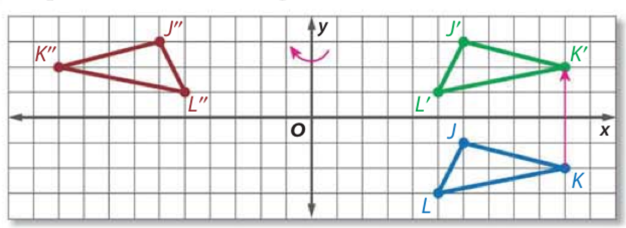
Step 1 translation along ⟨0, 4⟩

(x, y)	→	$(x, y + 4)$
$J(6, -1)$	→	$J'(6, 3)$
$K(10, -2)$	→	$K'(10, 2)$
$L(5, -3)$	→	$L'(5, 1)$

Step 2 reflection in the *y*-axis

(x, y)	→	$(-x, y)$
$J'(6, 3)$	→	$J''(-6, 3)$
$K'(10, 2)$	→	$K''(-10, 2)$
$L'(5, 1)$	→	$L''(-5, 1)$

Step 3 Graph △*JKL* and its image △*J″K″L″*.



GuidedPractice

Triangle *PQR* has vertices *P*(1, 1), *Q*(2, 5), and *R*(4, 2). Graph △*PQR* and its image after the indicated glide reflection.

1A. Translation: along ⟨−2, 0⟩
Reflection: in *x*-axis

1B. Translation: along ⟨−3, −3⟩
Reflection: in $y = x$

In Example 1, $\triangle JKL \cong \triangle J'K'L'$ and $\triangle J'K'L' \cong \triangle J''K''L''$. By the Transitive Property of Congruence, $\triangle JKL \cong \triangle J''K''L''$. This suggests the following theorem.

Theorem 9.1 Composition of Isometries

The composition of two (or more) isometries is an isometry.

You will prove one case of Theorem 9.1 in Exercise 30.

StudyTip

Rigid Motions Glide reflections, reflections, translations, and rotations are the only four *rigid motions* or isometries in a plane.

So, the composition of two or more isometries—reflections, translations, or rotations—results in an image that is congruent to its preimage.

Example 2 Graph Other Compositions of Isometries

The endpoints of $\overline{CD}$ are $C(-7, 1)$ and $D(-3, 2)$. Graph $\overline{CD}$ and its image after a reflection in the x-axis and a rotation 90° about the origin.

Step 1 reflection in the x-axis

$(x, y) \rightarrow (x, -y)$

$C(-7, 1) \rightarrow C'(-7, -1)$

$D(-3, 2) \rightarrow D'(-3, -2)$

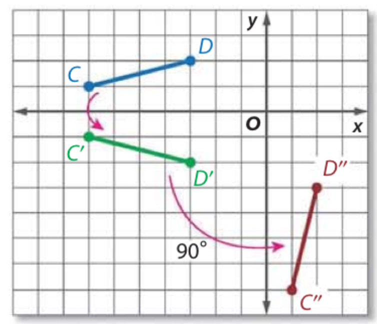
Step 2 rotation 90° about origin

$(x, y) \rightarrow (-y, x)$

$C'(-7, -1) \rightarrow C''(1, -7)$

$D'(-3, -2) \rightarrow D''(2, -3)$

Step 3 Graph $\overline{CD}$ and its image $\overline{C''D''}$.



ReadingMath

Double Primes Double primes are used to indicate that a vertex is the image of a second transformation.

GuidedPractice

Triangle ABC has vertices $A(-6, -2)$, $B(-5, -5)$, and $C(-2, -1)$. Graph $\triangle ABC$ and its image after the composition of transformations in the order listed.

2A. Translation: along $\langle 3, -1\rangle$
Reflection: in y-axis

2B. Rotation: 180° about origin
Translation: along $\langle -2, 4\rangle$

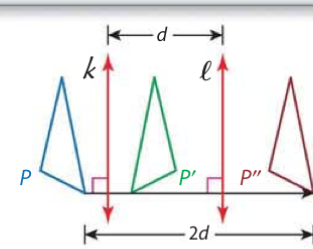
2 Compositions of Two Reflections

The composition of two reflections in parallel lines is the same as a translation.

Theorem 9.2 Reflections in Parallel Lines

The composition of two reflections in parallel lines can be described by a translation vector that is

- perpendicular to the two lines, and
- twice the distance between the two lines.



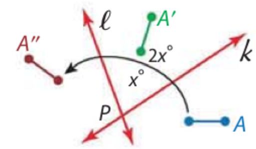
You will prove Theorem 9.2 in Exercise 36.

The composition of two reflections in intersecting lines is the same as a rotation.

Theorem 9.3 Reflections in Intersecting Lines

The composition of two reflections in intersecting lines can be described by a rotation

- about the point where the lines intersect and
- through an angle that is twice the measure of the acute or right angle formed by the lines.

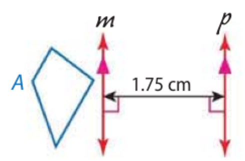


You will prove Theorem 9.3 in Exercise 37.

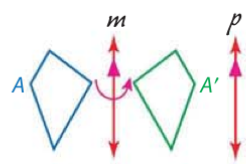
Example 3 Reflect a Figure in Two Lines

Copy and reflect figure *A* in line *m* and then line *p*. Then describe a single transformation that maps *A* onto *A*″.

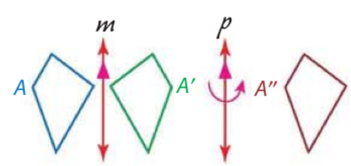
a.



Step 1 Reflect *A* in line *m*.

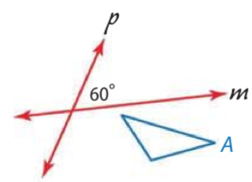


Step 2 Reflect *A*′ in line *p*.

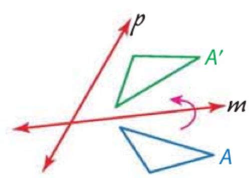


By Theorem 9.2, the composition of two reflections in parallel vertical lines *m* and *p* is equivalent to a horizontal translation right 2 • 1.75 or 3.5 centimeters.

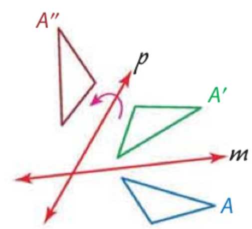
b.



Step 1



Step 2



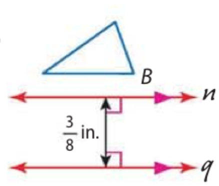
By Theorem 9.3, the composition of two reflections in intersecting lines *m* and *p* is equivalent to a 2 • 60° or 120° counterclockwise rotation about the point where lines *m* and *p* intersect.

> **WatchOut!**
>
> **Order of Composition** Be sure to compose two transformations according to the order in which they are given.

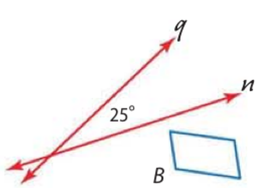
GuidedPractice

Copy and reflect figure *B* in line *n* and then line *q*. Then describe a single transformation that maps *B* onto *B*″.

3A.



3B.



Many patterns in the real world are created using compositions of transformations.

Real-WorldLink

In carpets, border patterns result when any of several basic transformations are repeated in one direction. There are seven possible combinations: translations, horizontal reflections, vertical reflections, vertical followed by horizontal reflections, glide reflections, rotations, and reflections followed by glide reflections.

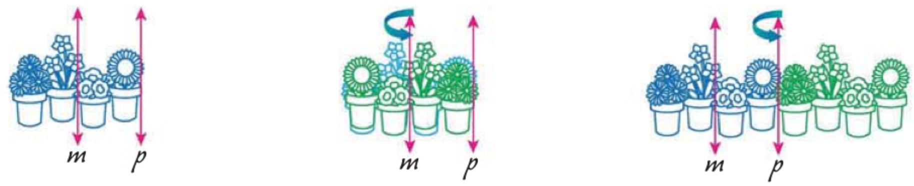
Source: The Textile Museum

Real-World Example 4 Describe Transformations

STATIONERY BORDERS Describe the transformations that are combined to create each stationery border shown.

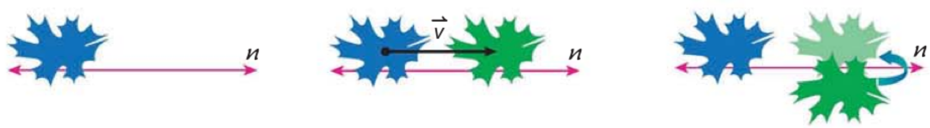
a.

The pattern is created by successive translations of the first four potted plants. So this pattern can be created by combining two reflections in lines m and p as shown. Notice that line m goes through the center of the preimage.



b.

The pattern is created by glide reflection. So this pattern can be created by combining a translation along translation vector $\vec{v}$ followed by a reflection over horizontal line n as shown.

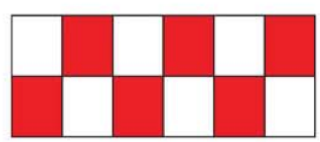


GuidedPractice

4. CARPET PATTERNS Describe the transformations that are combined to create each carpet pattern shown.

A.

B.



Martin Child/Photodisc/Getty Images

ConceptSummary Compositions of Translations

Glide Reflection	Translation	Rotation
the composition of a reflection and a translation	the composition of two reflections in parallel lines	the composition of two reflections in intersecting lines

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Triangle *CDE* has vertices $C(-5, -1)$, $D(-2, -5)$, and $E(-1, -1)$. Graph $\triangle CDE$ and its image after the indicated glide reflection.**

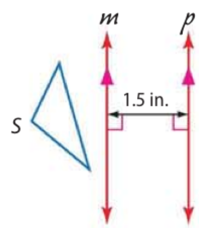
1. Translation: along $\langle 4, 0 \rangle$
Reflection: in *x*-axis

2. Translation: along $\langle 0, 6 \rangle$
Reflection: in *y*-axis

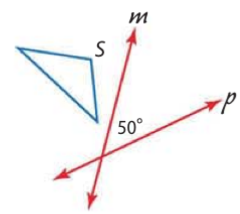
Example 2 **3.** The endpoints of $\overline{JK}$ are $J(2, 5)$ and $K(6, 5)$. Graph $\overline{JK}$ and its image after a reflection in the *x*-axis and a rotation 90° about the origin.

Example 3 **Copy and reflect figure *S* in line *m* and then line *p*. Then describe a single transformation that maps *S* onto *S″*.**

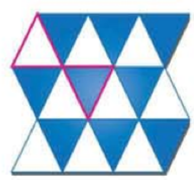
4.



5.



Example 4 **6.** **TILE PATTERNS** Viviana is creating a pattern for the top of a table with tiles in the shape of isosceles triangles. Describe the transformation combination that was used to transform the white triangle to the blue triangle.



Practice and Problem Solving

Extra Practice is on page R9.

Example 1 **Graph each figure with the given vertices and its image after the indicated glide reflection.**

7. $\triangle RST$: $R(1, -4)$, $S(6, -4)$, $T(5, -1)$
Translation: along $\langle 2, 0 \rangle$
Reflection: in *x*-axis

8. $\triangle JKL$: $J(1, 3)$, $K(5, 0)$, $L(7, 4)$
Translation: along $\langle -3, 0 \rangle$
Reflection: in *x*-axis

9. $\triangle XYZ$: $X(-7, 2)$, $Y(-5, 6)$, $Z(-2, 4)$
Translation: along $\langle 0, -1 \rangle$
Reflection: in *y*-axis

10. $\triangle ABC$: $A(2, 3)$, $B(4, 7)$, $C(7, 2)$
Translation: along $\langle 0, 4 \rangle$
Reflection: in *y*-axis

11. $\triangle DFG$: $D(2, 8)$, $F(1, 2)$, $G(4, 6)$
Translation: along $\langle 3, 3 \rangle$
Reflection: in $y = x$

12. $\triangle MPQ$: $M(-4, 3)$, $P(-5, 8)$, $Q(-1, 6)$
Translation: along $\langle -4, -4 \rangle$
Reflection: in $y = x$

Example 2 CCSS **SENSE-MAKING** **Graph each figure with the given vertices and its image after the indicated composition of transformations.**

13. $\overline{WX}$: $W(-4, 6)$ and $X(-4, 1)$
Reflection: in *x*-axis
Rotation: 90° about origin

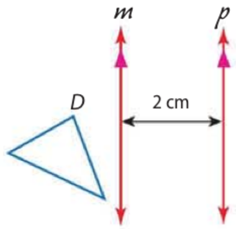
14. $\overline{AB}$: $A(-3, 2)$ and $B(3, 8)$
Rotation: 90° about origin
Translation: along $\langle 4, 4 \rangle$

15. $\overline{FG}$: $F(1, 1)$ and $G(6, 7)$
Reflection: in *x*-axis
Rotation: 180° about origin

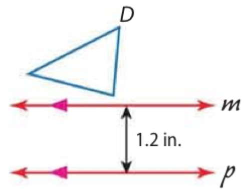
16. $\overline{RS}$: $R(2, -1)$ and $S(6, -5)$
Translation: along $\langle -2, -2 \rangle$
Reflection: in *y*-axis

Example 3 **Copy and reflect figure D in line m and then line p. Then describe a single transformation that maps D onto D''.**

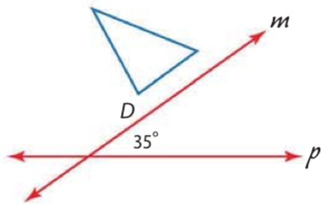
17.



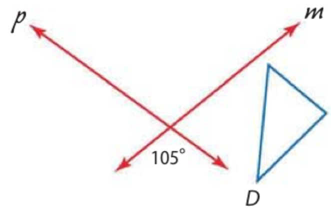
18.



19.



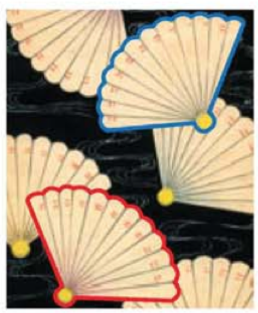
20.



Example 4 CCSS **MODELING** **Describe the transformations combined to create the outlined kimono fabric pattern.**

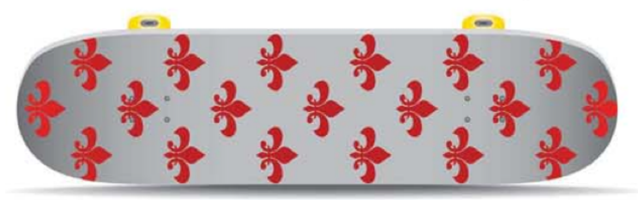
21.

22.



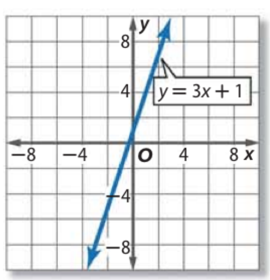
23.

24. **SKATEBOARDS** Elizabeth has airbrushed the pattern shown onto her skateboard. What combination of transformations did she use to create the pattern?

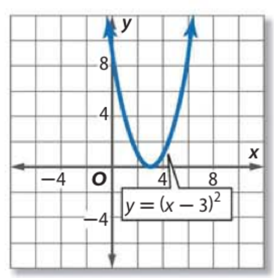


ALGEBRA **Graph each figure and its image after the indicated transformations.**

25. Rotation: 90° about the origin
Reflection: in x-axis



26. Reflection: in x-axis
Reflection: in y-axis

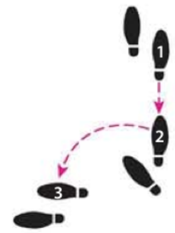


27. Find the coordinates of $\triangle A''B''C''$ after a reflection in the x-axis and a rotation of 180° about the origin if $\triangle ABC$ has vertices $A(-3, 1)$, $B(-2, 3)$, and $C(-1, 0)$.

PoodlesRock/Corbis

28. FIGURE SKATING Kayla is practicing her figure skating routine. What combination of transformations is needed for Kayla to start at A, skate to A', and end up at A''?

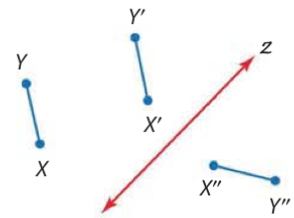
29. DANCING Describe the transformations combined to go from Step 1 to Step 3.



30. PROOF Write a paragraph proof for one case of the Composition of Isometries Theorem.

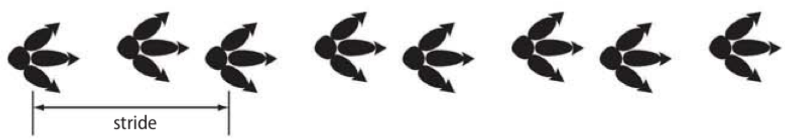
Given: A translation along $\langle a, b\rangle$ maps X to X' and Y to Y'. A reflection in z maps X' to X'' and Y' to Y''.

Prove: $\overline{XY} \cong \overline{X''Y''}$

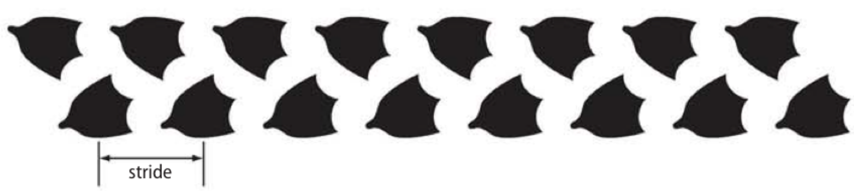


CCSS MODELING The length of an animal's stride is the distance between two consecutive tracks. The average stride length of a turkey is about 11 inches, and the average stride length of a duck is about 5 inches. Write a glide reflection that can be used to predict the location of the next track for each set of animal tracks.

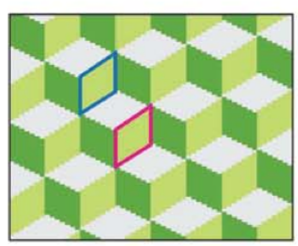
31. turkey



32. duck

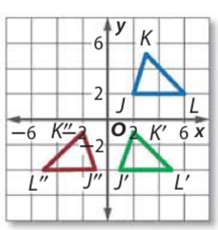


33. KNITTING Tonisha is knitting a scarf using the tumbling blocks pattern shown at the right. Describe the transformations combined to transform the red figure to the blue figure.

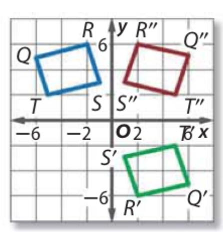


Describe the transformations that combined to map each figure.

34.



35.

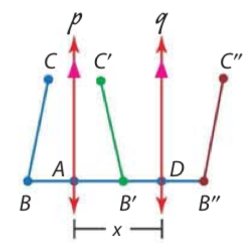


36. PROOF Write a two-column proof of Theorem 9.2.

Given: A reflection in line p maps $\overline{BC}$ to $\overline{B'C'}$.
A reflection in line q maps $\overline{B'C'}$ to $\overline{B''C''}$.
$p \parallel q$, $AD = x$

Prove: **a.** $\overline{BB''} \perp p$, $\overline{BB''} \perp q$

b. $BB'' = 2x$

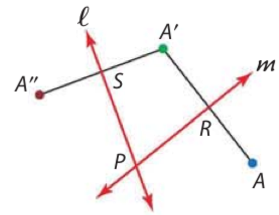


37. PROOF Write a paragraph proof of Theorem 9.3.

Given: Lines ℓ and m intersect at point P.
A is any point not on ℓ or m.

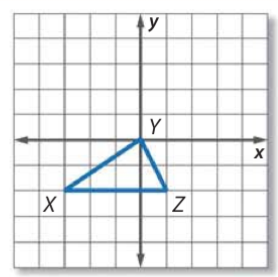
Prove: **a.** If you reflect point A in m, and then reflect its image A' in ℓ, A'' is the image of A after a rotation about point P.

b. $m\angle APA'' = 2(m\angle SPR)$



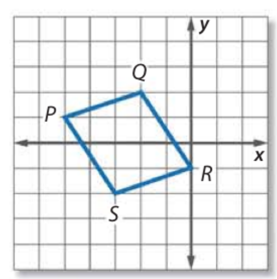
H.O.T. Problems Use Higher-Order Thinking Skills

38. ERROR ANALYSIS Daniel and Lolita are translating $\triangle XYZ$ along $\langle 2, 2\rangle$ and reflecting it in the line $y = 2$. Daniel says that the transformation is a glide reflection. Lolita disagrees and says that the transformation is a composition of transformations. Is either of them correct? Explain your reasoning.



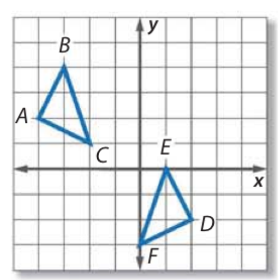
39. WRITING IN MATH Do any points remain invariant under glide reflections? under compositions of transformations? Explain.

40. CHALLENGE If $PQRS$ is translated along $\langle 3, -2\rangle$, reflected in $y = -1$, and rotated 90° about the origin, what are the coordinates of $P'''Q'''R'''S'''$?



41. CCSS ARGUMENTS If an image is to be reflected in the line $y = x$ and the x-axis, does the order of the reflections affect the final image? Explain.

42. OPEN ENDED Write a glide reflection or composition of transformations that can be used to transform $\triangle ABC$ to $\triangle DEF$.

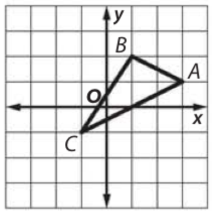


43. REASONING When two rotations are performed on a single image, does the order of the rotations *sometimes*, *always*, or *never* affect the location of the final image? Explain.

44. WRITING IN MATH Compare and contrast glide reflections and compositions of transformations.

Standardized Test Practice

45. $\triangle ABC$ is translated along the vector $\langle -2, 3 \rangle$ and then reflected in the x-axis. What are the coordinates of A' after the transformation?



A $(1, -4)$
B $(1, 4)$
C $(-1, 4)$
D $(-1, -4)$

46. SHORT RESPONSE What are the coordinates of D'' if $\overline{CD}$ with vertices $C(2, 4)$ and $D(8, 7)$ is translated along $\langle -6, 2 \rangle$ and then reflected over the y-axis?

47. ALGEBRA Write $\frac{18x^2 - 2}{3x^2 - 5x - 2}$ in simplest terms.

F $\frac{18}{3x + 1}$
G $\frac{2(3x + 1)}{x - 2}$
H $\frac{2(3x - 1)}{x - 2}$
J $2(3x - 1)$

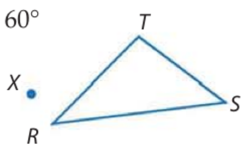
48. SAT/ACT If $f(x) = x^3 - x^2 - x$, what is the value of $f(-3)$?

A -39
B -33
C -21
D -15
E -12

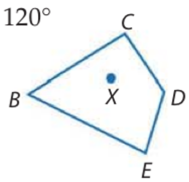
Spiral Review

Copy each polygon and point X. Then use a protractor and ruler to draw the specified rotation of each figure about point X. (Lesson 9-3)

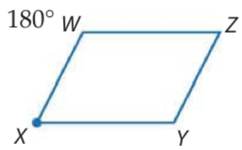
49. 60°



50. 120°



51. 180°



Graph each figure and its image along the given vector. (Lesson 9-2)

52. $\triangle FGH$ with vertices $F(1, -4)$, $G(3, -1)$, and $H(7, -1)$; $\langle 2, 6 \rangle$

53. quadrilateral $ABCD$ with vertices $A(-2, 7)$, $B(-1, 4)$, $C(2, 3)$, and $D(2, 7)$; $\langle -3, -5 \rangle$

54. AVIATION A jet is flying northwest, and its velocity is represented by $\langle -450, 450 \rangle$ miles per hour. The wind is from the west, and its velocity is represented by $\langle 100, 0 \rangle$ miles per hour. (Lesson 8-7)

a. Find the resultant vector for the jet in component form.

b. Find the magnitude of the resultant.

c. Find the direction of the resultant.

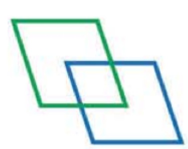
Skills Review

Each figure shows a preimage and its reflected image in some line. Copy each figure and draw the line of reflection.

55.

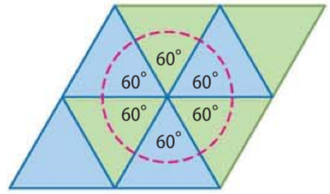
56.

57.



EXTEND 9-4 Geometry Lab
Tessellations

A **tessellation** is a pattern of one or more figures that covers a plane so that there are no overlapping or empty spaces. The sum of the angles around the vertex of a tessellation is 360°.



A **regular tessellation** is formed by only one type of regular polygon. A regular polygon will tessellate if it has an interior angle measure that is a factor of 360. A **semi-regular tessellation** is formed by two or more regular polygons.

Activity 1 Regular Tessellation

Determine whether each regular polygon will tessellate in the plane. Explain.

a. hexagon

Let x represent the measure of an interior angle of a regular hexagon.

$x = \frac{180(n-2)}{n}$ Interior Angle Formula

$= \frac{180(6-2)}{6}$ $n = 6$

$= 120$ Simplify.

Since 120 is a factor of 360, a regular hexagon will tessellate in the plane.

b. decagon

Let x represent the measure of an interior angle of a regular decagon.

$x = \frac{180(n-2)}{n}$ Interior Angle Formula

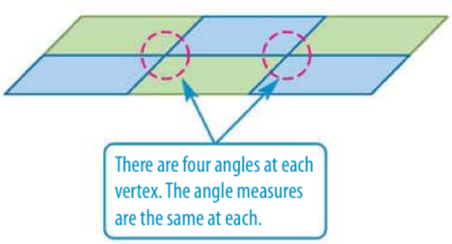
$= \frac{180(10-2)}{10}$ $n = 10$

$= 144$ Simplify.

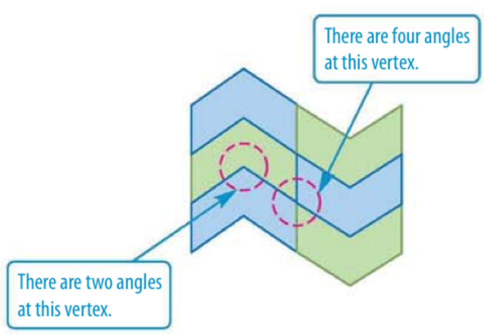
Since 144 is not a factor of 360, a regular decagon will not tessellate in the plane.

A tessellation is **uniform** if it contains the same arrangement of shapes and angles at each vertex.

Uniform



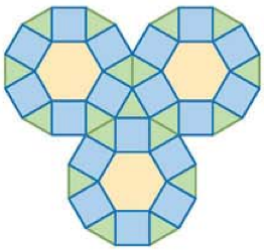
Not Uniform



Activity 2 Classify Tessellations

Determine whether each pattern is a tessellation. If so, describe it as *regular*, *semi-regular*, or *neither* and *uniform* or *not uniform*.

a.

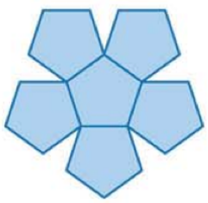


There is no unfilled space, and none of the figures overlap, so the pattern is a **tessellation**.

The tessellation consists of regular hexagons, squares and equilateral triangles, so it is **semi-regular**.

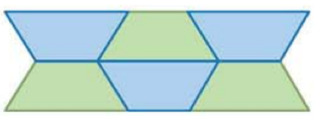
There are four angles around some of the vertices and five around others, so it is **not uniform**.

b.



There is unfilled space, so the pattern is a **not a tessellation**.

c.



There is no unfilled space, and none of the figures overlap, so the pattern is a **tessellation**.

The tessellation consists of trapezoids, which are not regular polygons, so it is **neither** regular nor semi-regular.

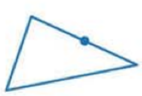
There are four angles around each of the vertices and the angle measures are the same at each vertex, so it is **uniform**.

You can use the properties of tessellations to design and create tessellations.

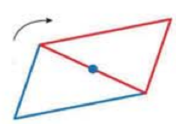
Activity 3 Draw a Tessellation

Draw a triangle and use it to create a tessellation.

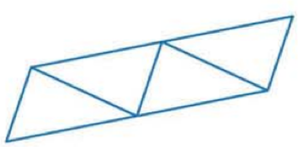
Step 1 Draw a triangle and find the midpoint of one side.



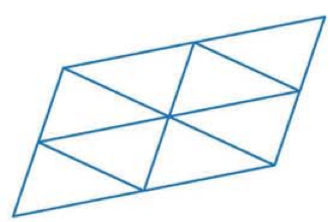
Step 2 Rotate the triangle 180° about the point.



Step 3 Translate the pair of triangles to make a row.



Step 4 Translate the row to make a tessellation.



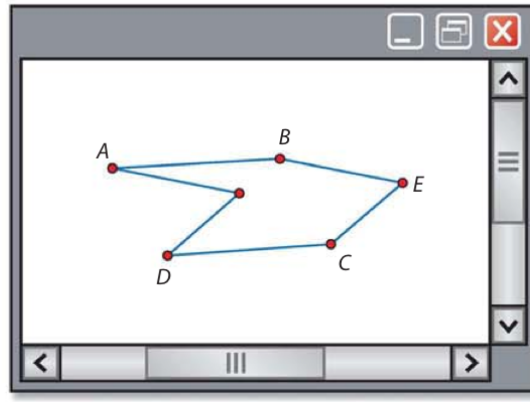
Geometry Lab

Tessellations *Continued*

Activity 4 Tessellations using Technology

Use Geometer's Sketchpad to create a tessellation.

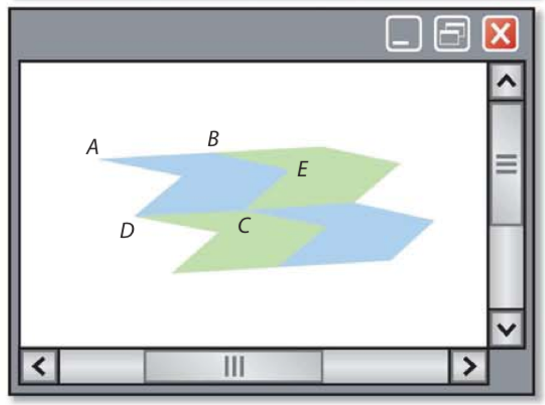
Step 1 Insert three points and construct a line through two of the points. Then construct the line parallel to the first line through the third point using the **Parallel Line** option from the **Construct** menu. Complete the parallelogram and label the points *A*, *B*, *C*, and *D*. Hide the lines.



Step 2 Insert another point *E* on the exterior of the parallelogram. Draw the segments between *A* and *B*, *B* and *E*, *E* and *C*, and *C* and *D*.

Step 3 Highlight *B* and then *A*. From the **Transform** menu, choose **Mark Vector**. Select the $\overline{BE}$, $\overline{EC}$, and point *E*. From the **Transform** menu, choose **Translate**.

Step 4 Starting with *A*, select all of the vertices around the perimeter of the polygon. Choose **Hexagon Interior** from the **Construct** menu.



Step 5 Choose point *A* and then point *B* and mark the vector as you did in Step 3. Select the interior of the polygon and choose **Translate** from the **Transform** menu. Continue the tessellation by marking vectors and translating the polygon. You can choose **Color** from the **Display** menu to create a color pattern.

Exercises

Determine whether each regular polygon will tessellate in the plane. Write *yes* or *no*. Explain.

1. triangle

2. pentagon

3. 16-gon

Determine whether each pattern is a tessellation. Write *yes* or *no*. If so, describe it as *regular*, *semi-regular*, or *neither* and *uniform* or *not uniform*.

4.


5.


6.


Draw a tessellation using the following shape(s).

7. octagon and square

8. hexagon and triangle

9. right triangle

10. trapezoid and a parallelogram

11. **WRITING IN MATH** Find examples of the use of tessellations in architecture, mosaics, and artwork. For each example, explain how tessellations were used.

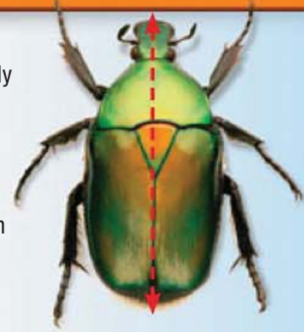
12. **MAKE A CONJECTURE** Describe a figure that you think will tessellate in three-dimensional space. Explain your reasoning.

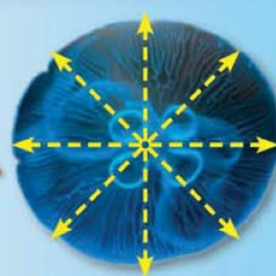
LESSON 9-5 Symmetry

Then
- You drew reflections and rotations of figures.

Now
1. Identify line and rotational symmetries in two-dimensional figures.
2. Identify plane and axis symmetries in three-dimensional figures.

Why?
- In the animal kingdom, the symmetry of an animal's body is often an indication of the animal's complexity. Animals displaying line symmetry, such as insects, are usually more complex life forms than those displaying rotational symmetry, like a jellyfish.





NewVocabulary
symmetry
line symmetry
line of symmetry
rotational symmetry
center of symmetry
order of symmetry
magnitude of symmetry
plane symmetry
axis symmetry

Common Core State Standards

Content Standards

G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Mathematical Practices

4 Model with mathematics.

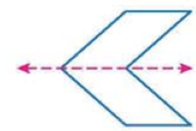
8 Look for and express regularity in repeated reasoning.

1 Symmetry in Two-Dimensional Figures

A figure has **symmetry** if there exists a rigid motion—reflection, translation, rotation, or glide reflection—that maps the figure onto itself. One type of symmetry is line symmetry.

KeyConcept Line Symmetry

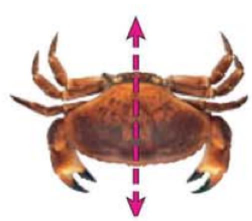
A figure in the plane has **line symmetry** (or *reflection symmetry*) if the figure can be mapped onto itself by a reflection in a line, called a **line of symmetry** (or *axis of symmetry*).



Real-World Example 1 Identify Line Symmetry

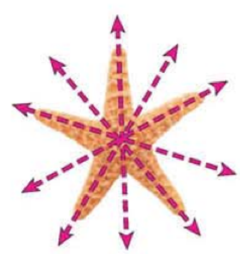
BEACHES **State whether the object appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.**

a.



Yes; the crab has one line of symmetry.

b.



Yes; the starfish has five lines of symmetry.

c.

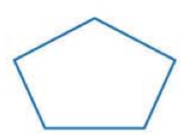
No; there is no line in which the oyster shell can be reflected so that it maps onto itself.

GuidedPractice

State whether the figure has line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

1A.

1B.



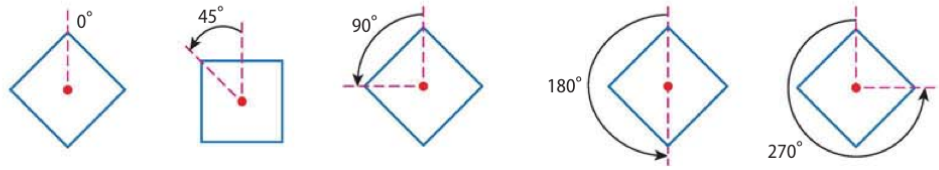
1C.

Another type of symmetry is rotational symmetry.

KeyConcept Rotational Symmetry

A figure in the plane has **rotational symmetry** (or *radial symmetry*) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure, called the **center of symmetry** (or *point of symmetry*).

Examples The figure below has rotational symmetry because a rotation of 90°, 180°, or 270° maps the figure onto itself.



The number of times a figure maps onto itself as it rotates from 0° to 360° is called the **order of symmetry**. The **magnitude of symmetry** (or angle of rotation) is the smallest angle through which a figure can be rotated so that it maps onto itself. The order and magnitude of a rotation are related by the following equation.

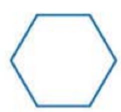
$$\text{magnitude} = 360° \div \text{order}$$

The figure above has rotational symmetry of order 4 and magnitude 90°.

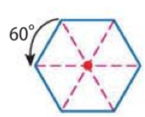
Example 2 Identify Rotational Symmetry

State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

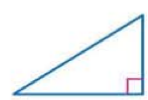
a.



Yes; the regular hexagon has order 6 rotational symmetry and magnitude 360° ÷ 6 or 60°. The center is the intersection of the diagonals.



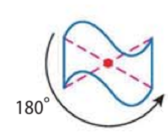
b.



No; no rotation between 0° and 360° maps the right triangle onto itself.

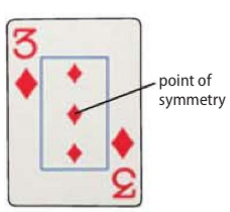
c.

Yes; the figure has order 2 rotational symmetry and magnitude 360° ÷ 2 or 180°. The center is the intersection of the diagonals.



StudyTip

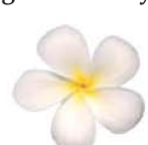
Point Symmetry A figure has *point symmetry* if the figure can be mapped onto itself by a rotation of 180°. A playing card exhibits point symmetry. It looks the same right-side up as upside down.



GuidedPractice

FLOWERS **State whether the flower appears to have rotational symmetry. Write *yes* or *no*. If so, copy the flower, locate the center of symmetry, and state the order and magnitude of symmetry.**

2A.



2B.

2C.

(bl)Mark Ransom Photography, (bc)Organica/Alamy, (br)CuboImages srl/Alamy

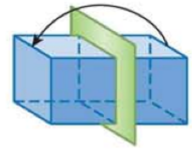
2 Symmetry in Three-Dimensional Figures

Three-dimensional figures can also have symmetry.

KeyConcept Three-Dimensional Symmetries

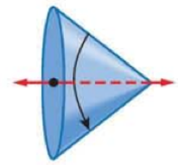
Plane Symmetry

A three-dimensional figure has **plane symmetry** if the figure can be mapped onto itself by a reflection in a plane.



Axis Symmetry

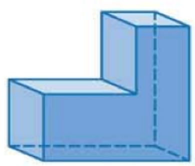
A three-dimensional figure has **axis symmetry** if the figure can be mapped onto itself by a rotation between 0° and 360° in a line.



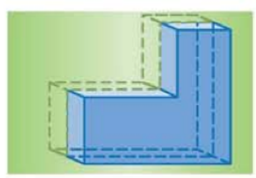
Example 3 Three-Dimensional Symmetry

State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

a. L-shaped prism

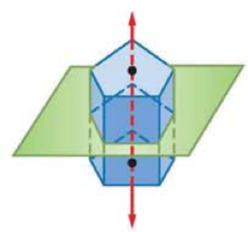


plane symmetry



b. regular pentagonal prism

both plane symmetry and axis symmetry



ReviewVocabulary

prism a polyhedron with two parallel congruent bases connected by parallelogram faces

Real-WorldLink

Aerodynamically designed to spin after it is thrown, a football's shape is that of a prolate spheroid. This means that one axis of symmetry is longer than its other axes.

Source: *Complete Idiot's Guide to Football*

GuidedPractice

SPORTS **State whether each piece of sports equipment appears to have *plane* symmetry, *axis* symmetry, *both*, or *neither* (ignoring the equipment's stitching or markings).**

3A.

3B.

3C.

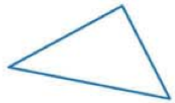
3D.

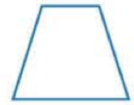
Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

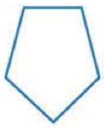
Example 1 **State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.**

1. 

2. 

3. 

Example 2 **State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.**

4. 

5. 

6. 

Examples 2–3 **7** **U.S. CAPITOL** Completed in 1863, the dome is one of the most recent additions to the United States Capitol. It is supported by 36 iron ribs and has 108 windows, divided equally among three levels.

a. Excluding the spire of the dome, how many horizontal and vertical planes of symmetry does the dome appear to have?

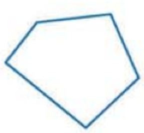
b. Does the dome have axis symmetry? If so, state the order and magnitude of symmetry.

Example 3 **8.** State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

Practice and Problem Solving

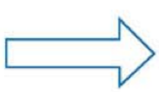
Extra Practice is on page R9.

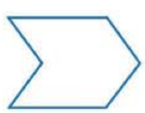
Example 1 **CCSS REGULARITY** **State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.**

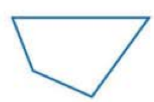
9. 

10. 

11. 

12. 

13. 

14. 

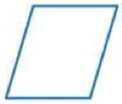
FLAGS **State whether each flag design appears to have line symmetry. Write *yes* or *no*. If so, copy the flag, draw all lines of symmetry, and state their number.**

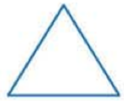
15. 

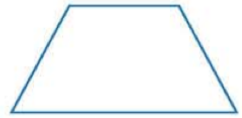
16. 

17. 

Example 2 **State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.**

18. 

19. 

20. 

21. 

22. 

23. 

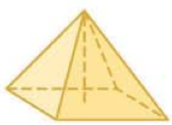
Example 2 **WHEELS** **State whether each wheel cover appears to have rotational symmetry. Write *yes* or *no*. If so, state the order and magnitude of symmetry.**

24. 

25. 

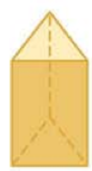
26. 

Example 3 **State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.**

27. 

28. 

29. 

30. 

CONTAINERS **Determine the number of horizontal and vertical planes of symmetry for each container shown below.**

31. 

32. 

33. 

34. **CCSS MODELING** Symmetry is an important component of photography. Photographers often use reflection in water to create symmetry in photos. The photo at the right is a long exposure shot of the Eiffel tower reflected in a pool.

a. Describe the two-dimensional symmetry created by the photo.

b. Is three-dimensional symmetry applicable? Explain your reasoning.

COORDINATE GEOMETRY **Determine whether the figure with the given vertices has *line* symmetry and/or *rotational* symmetry.**

35. $A(-4, 0), B(0, 4), C(4, 0), D(0, -4)$

36. $R(-3, 3), S(-3, -3), T(3, 3)$

37. $F(0, -4), G(-3, -2), H(-3, 2), J(0, 4), K(3, 2), L(3, -2)$

38. $W(-2, 3), X(-3, -3), Y(3, -3), Z(2, 3)$

ALGEBRA **Graph the function and determine whether the graph has *line* and/or *rotational* symmetry. If so, state the order and magnitude of symmetry, and write the equations of any lines of symmetry.**

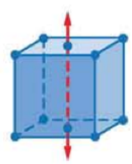
39. $y = x$

40. $y = x^2 + 1$

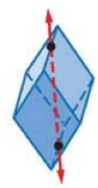
41. $y = -x^3$

CRYSTALLOGRAPHY **Determine whether the crystals below have *plane* symmetry and/or *axis* symmetry. If so, state the magnitude of symmetry.**

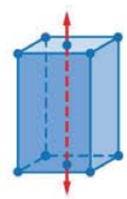
42.



43.



44.



45. **MULTIPLE REPRESENTATIONS** In this problem, you will use dynamic geometric software to investigate line and rotational symmetry in regular polygons.

   a. **Geometric** Use The Geometer's Sketchpad to draw an equilateral triangle. Use the reflection tool under the transformation menu to investigate and determine all possible lines of symmetry. Then record their number.

   b. **Geometric** Use the rotation tool under the transformation menu to investigate the rotational symmetry of the figure in part **a**. Then record its order of symmetry.

   c. **Tabular** Repeat the process in parts **a** and **b** for a square, regular pentagon, and regular hexagon. Record the number of lines of symmetry and the order of symmetry for each polygon.

   d. **Verbal** Make a conjecture about the number of lines of symmetry and the order of symmetry for a regular polygon with n sides.

H.O.T. Problems Use Higher-Order Thinking Skills

46. **CCSS CRITIQUE** Jaime says that Figure A has only line symmetry, and Jewel says that Figure A has only rotational symmetry. Is either of them correct? Explain your reasoning.

Figure A

47. **CHALLENGE** A quadrilateral in the coordinate plane has exactly two lines of symmetry, $y = x - 1$ and $y = -x + 2$. Find possible vertices for the figure. Graph the figure and the lines of symmetry.

48. **REASONING** A regular polyhedron has axis symmetry of order 3, but does not have plane symmetry. What is the figure? Explain.

49. **OPEN ENDED** Draw a figure with line symmetry but not rotational symmetry. Explain.

50. **WRITING IN MATH** How are line symmetry and rotational symmetry related?

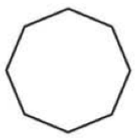
Standardized Test Practice

51. How many lines of symmetry can be drawn on the picture of the Canadian flag below?

A 0 **C** 2

B 1 **D** 4

52. GRIDDED RESPONSE What is the order of symmetry for the figure below?



53. ALGEBRA A computer company ships computers in wooden crates that each weigh 45 pounds when empty. If each computer weighs no more than 13 pounds, which inequality *best* describes the total weight in pounds w of a crate of computers that contains c computers?

F $c \leq 13 + 45w$ **H** $w \leq 13c + 45$

G $c \geq 13 + 45w$ **J** $w \geq 13c + 45$

54. SAT/ACT What is the slope of the line determined by the linear equation $5x - 2y = 10$?

A -5 **D** $\frac{2}{5}$

B $-\frac{5}{2}$ **E** $\frac{5}{2}$

C $-\frac{2}{5}$

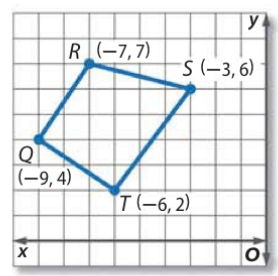
Spiral Review

Triangle *JKL* has vertices *J*(1, 5), *K*(3, 1), and *L*(5, 7). Graph △*JKL* and its image after the indicated transformation. (Lesson 9-4)

55. Translation: along ⟨−7, −1⟩
Reflection: in x-axis

56. Translation: along ⟨1, 2⟩
Reflection: in y-axis

57. Quadrilateral *QRST* is shown at the right. What is the image of point *R* after a rotation 180° counterclockwise about the origin? (Lesson 9-3)

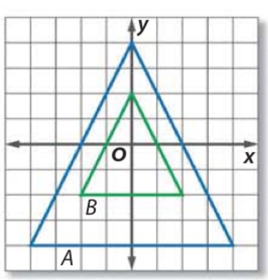


58. AMUSEMENT PARKS From the top of a roller coaster, 60 yards above the ground, a rider looks down and sees the merry-go-round and the Ferris wheel. If the angles of depression are 11° and 8° respectively, how far apart are the merry-go-round and the Ferris wheel? (Lesson 8-5)

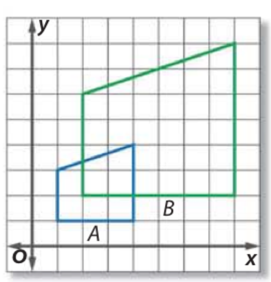
Skills Review

Determine whether the dilation from Figure A to Figure B is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

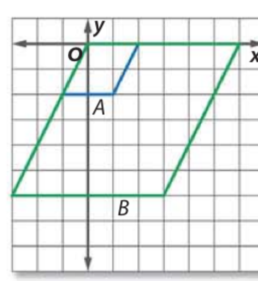
59.



60.



61.



EXTEND 9-5

Geometry Lab
Exploring Constructions with a Reflective Device

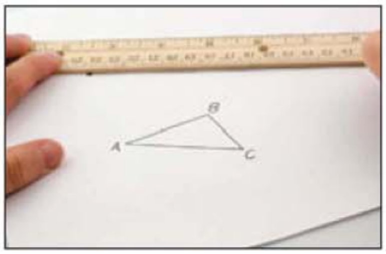
A reflective device is a tool made of semitransparent plastic that reflects objects. It works best if you lay it on a flat service in a well-lit room. You can use a reflective device to transform geometric objects.

CCSS Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

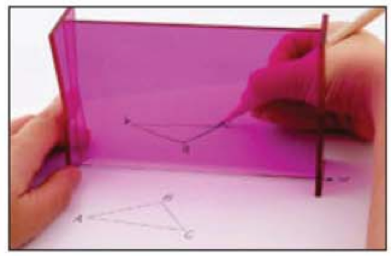
Activity 1 Reflect a Triangle

Use a reflective device to reflect $\triangle ABC$ in w. Label the reflection $\triangle A'B'C'$.

Step 1 Draw $\triangle ABC$ and the line of reflection w.



Step 2 With the reflective device on line w, draw points for the vertices of the reflection.



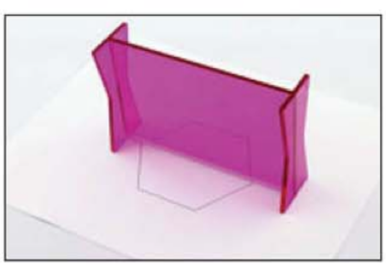
Step 3 Use a straightedge to connect the points to form $\triangle A'B'C'$.

We have used a compass, straightedge, string, and paper folding to make geometric constructions. You can also use a reflective device for constructions.

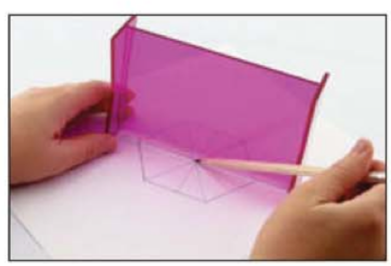
Activity 2 Construct Lines of Symmetry

Use a reflective device to construct the lines of symmetry for a regular hexagon.

Step 1 Draw a regular hexagon. Place the reflective device on the shape and move it until one half of the shape matches the reflection of the other half. Draw the line of symmetry.



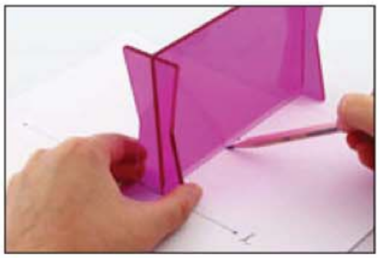
Step 2 Repeat Step 1 until you have found all the lines of symmetry.



Activity 3 Construct a Parallel line

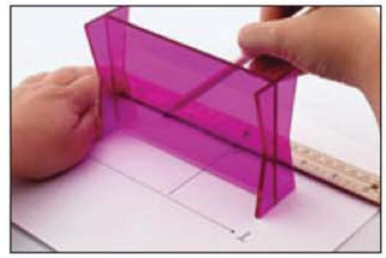
Use a reflective device to reflect line ℓ to line m that is parallel and passes through point P.

Step 1



Draw line ℓ and point P. Place a short side of the reflective device on line ℓ and the long side on point P. Draw a line. This line is perpendicular to ℓ through P.

Step 2

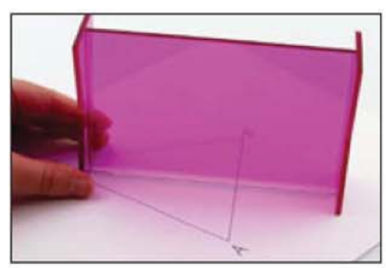


Place the reflective device so that the perpendicular line coincides with itself and the reflection of line ℓ passes through point P. Use a straightedge to draw the parallel line m through P.

In Explore Lesson 5-1, we constructed perpendicular bisectors with paper folding. You can also use a reflective device to construct perpendicular bisectors of a triangle.

Activity 4 Construct Perpendicular Bisectors

Use a reflective device to find the circumcenter of $\triangle ABC$.

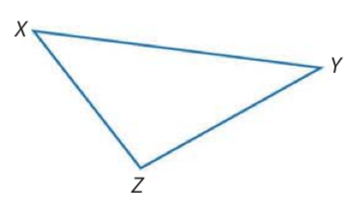


Step 1 Draw $\triangle ABC$. Place the reflective device between A and B and adjust it until A and B coincide. Draw the line of symmetry.

Step 2 Repeat Step 1 for sides $\overline{AC}$ and $\overline{BC}$. Then place a point at the intersection of the three perpendicular bisectors. This is the circumcenter of the triangle.

Model and Analyze

1. How do you know that the steps in Activity 4 give the actual perpendicular bisector and the circumcenter of $\triangle ABC$?



2. Construct the angle bisectors and find the incenter of $\triangle XYZ$. Describe how you used the reflective device for the construction.

EXPLORE 9-6

Graphing Technology Lab
Dilations

You can use TI-Nspire Technology to explore properties of dilations.

CCSS Common Core State Standards
Content Standards
G.SRT.1 Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations given by a center and a scale factor:
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
Mathematical Practices 5

Activity 1 Dilation of a Triangle

Dilate a triangle by a scale factor of 1.5.

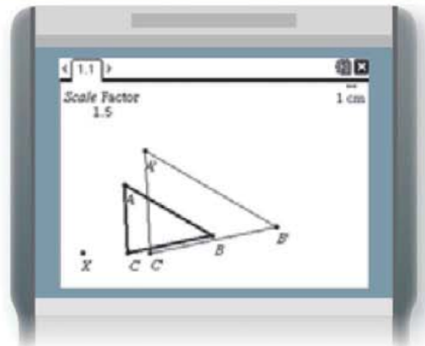
Step 1 Add a new **Geometry** page. Then, from the **Points & Lines** menu, use the **Point** tool to add a point and label it *X*.

Step 2 From the **Shapes** menu, select **Triangle** and specify three points. Label the points *A*, *B*, and *C*.

Step 3 From the **Actions** menu, use the **Text** tool to separately add the text *Scale Factor* and *1.5* to the page.

Step 4 From the **Transformation** menu, select **Dilation**. Then select point *X*, $\triangle ABC$, and the text *1.5*.

Step 5 Label the points on the image *A′*, *B′*, and *C′*.



Analyze the Results

1. Using the **Slope** tool on the **Measurement** menu, describe the effect of the dilation on $\overline{AB}$. That is, how are the lines through $\overline{AB}$ and $\overline{A'B'}$ related?
2. What is the effect of the dilation on the line passing through side $\overline{CA}$?
3. What is the effect of the dilation on the line passing through side $\overline{CB}$?

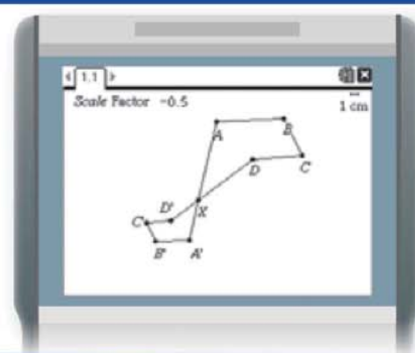
Activity 2 Dilation of a Polygon

Dilate a polygon by a scale factor of −0.5.

Step 1 Add a new **Geometry** page and draw polygon *ABCDX* as shown. Add the text *Scale Factor* and *−0.5* to the page.

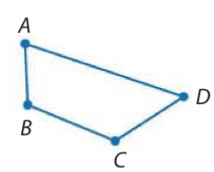
Step 2 From the **Transformation** menu, select **Dilation**. Then select point *X*, polygon *ABCDX*, and the text *−0.5*.

Step 3 Label the points on the image *A′*, *B′*, *C′*, and *D′*.



Model and Analyze

4. Analyze the effect of the dilation in Activity 2 on sides that contain the center of the dilation.
5. Analyze the effect of a dilation of trapezoid *ABCD* shown with a scale factor of 0.75 and the center of the dilation at *A*.



6. **MAKE A CONJECTURE** Describe the effect of a dilation on segments that pass through the center of a dilation and segments that do not pass through the center of a dilation.

Activity 3 Dilation of a Segment

Dilate a segment $\overline{AB}$ by the indicated scale factor.

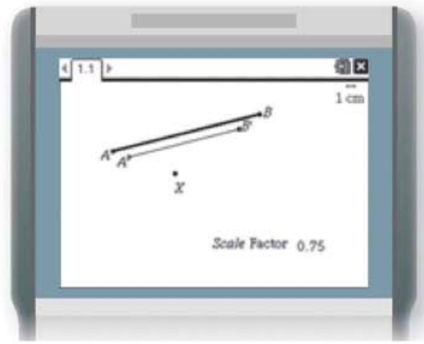
a. scale factor: 0.75

Step 1 On a new **Geometry** page, draw a line segment using the **Points & Lines** menu. Label the endpoints *A* and *B*. Then add and label a point *X*.

Step 2 Add the text *Scale Factor* and *0.75* to the page.

Step 3 From the **Transformation** menu, select **Dilation**. Then select point *X*, $\overline{AB}$, and the text *0.75*.

Step 4 Label the dilated segment $\overline{A'B'}$.

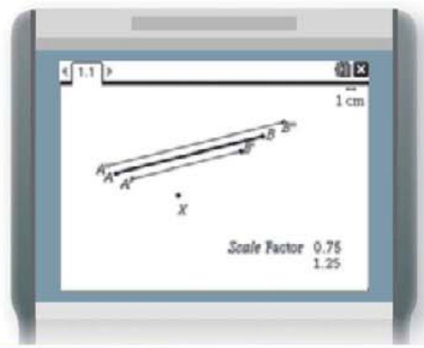


b. scale factor: 1.25

Step 1 Add the text *1.25* to the page.

Step 2 From the **Transformation** menu, select **Dilation**. Then select point *X*, $\overline{AB}$, and the text 1.25.

Step 3 Label the dilated segment $\overline{A''B''}$.



Model and Analyze

7. Using the **Length** tool on the **Measurement** menu, find the measures of $\overline{AB}$, $\overline{A'B'}$, and $\overline{A''B''}$.

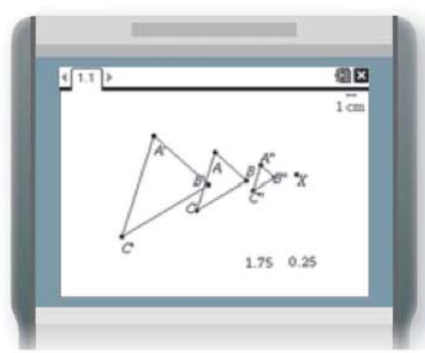
8. What is the ratio of $A'B'$ to AB? What is the ratio of $A''B''$ to AB?

9. What is the effect of the dilation with scale factor 0.75 on segment $\overline{AB}$? What is the effect of the dilation with scale factor 1.25 on segment $\overline{AB}$?

10. Dilate segment $\overline{AB}$ in Activity 3 by scale factors of −0.75 and −1.25. Describe the effect on the length of each dilated segment.

11. MAKE A CONJECTURE Describe the effect of a dilation on the length of a line segment.

12. Describe the dilation from $\overline{AB}$ to $\overline{A'B'}$ and $\overline{A'B'}$ to $\overline{A''B''}$ in the triangles shown.



LESSON 9-6 Dilations

Then

- You identified dilations and verified them as similarity transformations.

Now

1. Draw dilations.
2. Draw dilations in the coordinate plane.

Why?

- Some photographers still prefer traditional cameras and film to produce negatives. From these negatives, photographers can create scaled reproductions.

CCSS Common Core State Standards

Content Standards

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.SRT.1 Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

5 Use appropriate tools strategically.

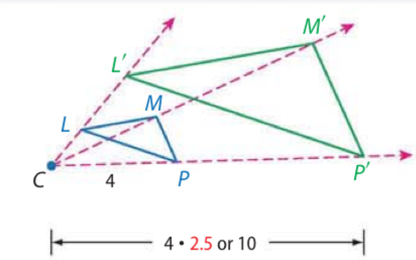
1 Draw Dilations

A dilation or *scaling* is a similarity transformation that enlarges or reduces a figure proportionally with respect to a *center* point and a *scale* factor.

KeyConcept Dilation

A dilation with center C and positive scale factor k, $k \neq 1$, is a function that maps a point P in a figure to its image such that

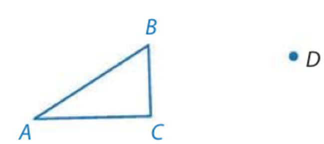
- if point P and C coincide, then the image and preimage are the same point, or
- if point P is not the center of dilation, then P' lies on $\overrightarrow{CP}$ and $CP' = k(CP)$.



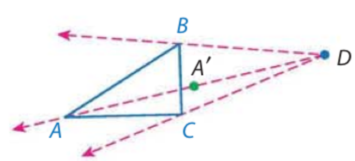
$\triangle L'M'P'$ is the image of $\triangle LMP$ under a dilation with center C and scale factor 2.5.

Example 1 Draw a Dilation

Copy $\triangle ABC$ and point D. Then use a ruler to draw the image of $\triangle ABC$ under a dilation with center D and scale factor $\frac{1}{2}$.

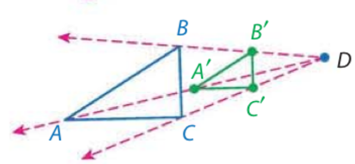


Step 1 Draw rays from D though each vertex.



Step 2 Locate A' on $\overrightarrow{DA}$ such that $DA' = \frac{1}{2}DA$.

Step 3 Locate B' on $\overrightarrow{DB}$ and C' on $\overrightarrow{DC}$ in the same way. Then draw $\triangle A'B'C'$.

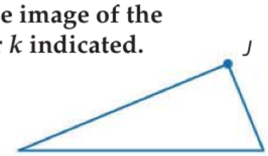


GuidedPractice

Copy the figure and point J. Then use a ruler to draw the image of the figure under a dilation with center J and the scale factor k indicated.

1A. $k = \frac{3}{2}$

1B. $k = 0.75$

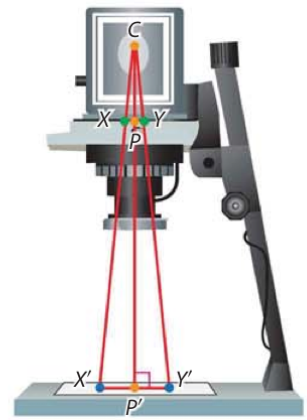


In Lesson 7-6, you also learned that if $k > 1$, then the dilation is an *enlargement*. If $0 < k < 1$, then the dilation is a *reduction*. Since $\frac{1}{2}$ is between 0 and 1, the dilation in Example 1 is a reduction.

A dilation with a scale factor of 1 is called an *isometry dilation*. It produces an image that coincides with the preimage. The two figures are congruent.

Real-World Example 2 Find the Scale Factor of a Dilation

PHOTOGRAPHY To create different-sized prints, you can adjust the distance between a film negative and the enlarged print by using a photographic enlarger. Suppose the distance between the light source C and the negative is 45 millimeters (CP). To what distance PP' should you adjust the enlarger to create a 22.75-centimeter wide print ($X'Y'$) from a 35-millimeter wide negative (XY)?



Understand This problem involves a dilation. The center of dilation is C, $XY = 35$ mm, $X'Y' = 22.75$ cm or 227.5 mm, and $CP = 45$ mm. You are asked to find PP'.

Plan Find the scale factor of the dilation from the preimage XY to the image $X'Y'$. Use the scale factor to find CP' and then use CP and CP' to find PP'.

Solve The scale factor k of the enlargement is the ratio of a length on the image to a corresponding length on the preimage.

$k = \frac{\text{image length}}{\text{preimage length}}$	Scale factor of image
$= \frac{X'Y'}{XY}$	image $= X'Y'$, preimage $= XY$
$= \frac{227.5}{35}$ or 6.5	Divide.

Use this scale factor of 6.5 to find CP'.

$CP' = k(CP)$	Definition of dilation
$= 6.5(45)$	$k = 6.5$ and $CP = 45$
$= 292.5$	Multiply.

Use CP' and CP to find PP'.

$CP + PP' = CP'$	Segment Addition
$45 + PP' = 292.5$	$CP = 45$ and $CP' = 292.5$
$PP' = 247.5$	Subtract 45 from each side.

So the enlarger should be adjusted so that the distance from the negative to the enlarged print (PP') is 247.5 millimeters or 24.75 centimeters.

Check Since the dilation is an enlargement, the scale factor should be greater than 1. Since $6.5 > 1$, the scale factor found is reasonable. ✓

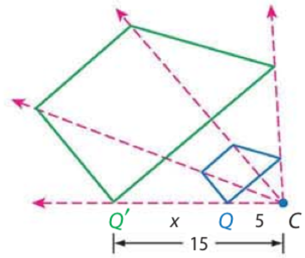
Problem-Solving Tip

CCSS Perseverance

To prevent careless errors in your calculations, estimate the answer to a problem before solving. In Example 2, you can estimate the scale factor of the dilation to be about $\frac{240}{40}$ or 6. Then CP' would be about $6 \cdot 50$ or 300 and PP' about $300 - 50$ or 250 millimeters, which is 25 centimeters. A measure of 24.75 centimeters is close to this estimate, so the answer is reasonable.

GuidedPractice

2. Determine whether the dilation from Figure Q to Q' is an *enlargement* or a *reduction*. Then find the scale factor of the dilation and x.



2 Dilations in the Coordinate Plane

You can use the following rules to find the image of a figure after a dilation centered at the origin.

StudyTip

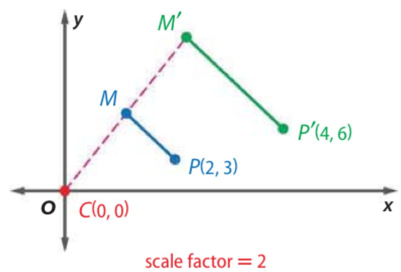
Negative Scale Factors Dilations can also have negative scale factors. You will investigate this type of dilation in Exercise 36.

KeyConcept Dilations in the Coordinate Plane

Words To find the coordinates of an image after a dilation centered at the origin, multiply the x- and y-coordinates of each point on the preimage by the scale factor of the dilation, k.

Symbols $(x, y) \rightarrow (kx, ky)$

Example



Example 3 Dilations in the Coordinate Plane

Quadrilateral $JKLM$ has vertices $J(-2, 4)$, $K(-2, -2)$, $L(-4, -2)$, and $M(-4, 2)$. Graph the image of $JKLM$ after a dilation centered at the origin with a scale factor of 2.5.

Multiply the x- and y-coordinates of each vertex by the scale factor, 2.5.

$(x, y) \rightarrow (2.5x, 2.5y)$

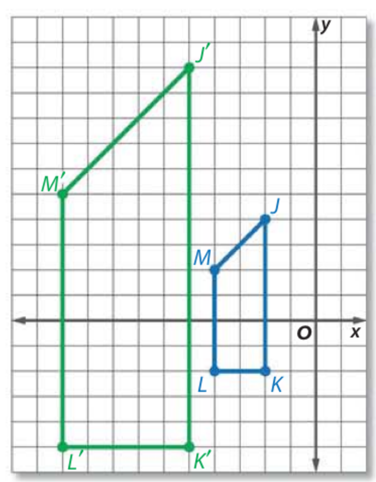
$J(-2, 4) \rightarrow J'(-5, 10)$

$K(-2, -2) \rightarrow K'(-5, -5)$

$L(-4, -2) \rightarrow L'(-10, -5)$

$M(-4, 2) \rightarrow M'(-10, 5)$

Graph $JKLM$ and its image $J'K'L'M'$.



GuidedPractice

Find the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

3A. $Q(0, 6)$, $R(-6, -3)$, $S(6, -3)$; $k = \frac{1}{3}$

3B. $A(2, 1)$, $B(0, 3)$, $C(-1, 2)$, $D(0, 1)$; $k = 2$

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

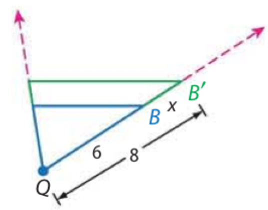
Example 1 **Copy the figure and point M. Then use a ruler to draw the image of the figure under a dilation with center M and the scale factor k indicated.**

1. $k = \frac{1}{4}$

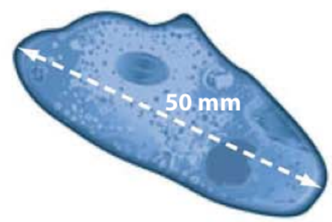
2. $k = 2$

Example 2

3 Determine whether the dilation from Figure B to B' is an *enlargement* or a *reduction*. Then find the scale factor of the dilation and x.



4. **BIOLOGY** Under a microscope, a single-celled organism 200 microns in length appears to be 50 millimeters long. If 1 millimeter = 1000 microns, what magnification setting (scale factor) was used? Explain your reasoning.



Example 3 **Graph the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.**

5. $W(0, 0), X(6, 6), Y(6, 0); k = 1.5$

6. $Q(-4, 4), R(-4, -4), S(4, -4), T(4, 4); k = \frac{1}{2}$

7. $A(-1, 4), B(2, 4), C(3, 2), D(-2, 2); k = 2$

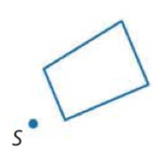
8. $J(-2, 0), K(2, 4), L(8, 0), M(2, -4); k = \frac{3}{4}$

Practice and Problem Solving

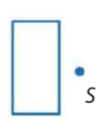
Extra Practice is on page R9.

Example 1 **CCSS TOOLS** **Copy the figure and point S. Then use a ruler to draw the image of the figure under a dilation with center S and the scale factor k indicated.**

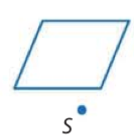
9. $k = \frac{5}{2}$



10. $k = 3$

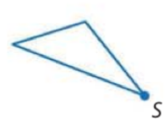


11. $k = 0.8$

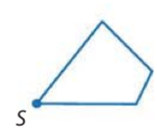


12. $k = \frac{1}{3}$

13. $k = 2.25$

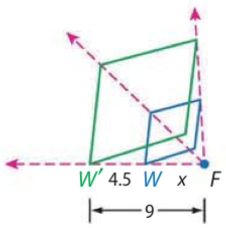


14. $k = \frac{7}{4}$

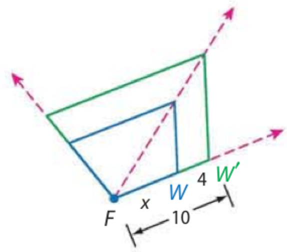


Example 2 **Determine whether the dilation from figure W to W' is an *enlargement* or a *reduction*. Then find the scale factor of the dilation and x.**

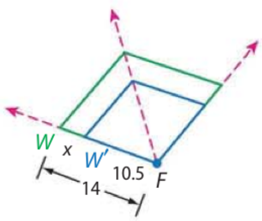
15.



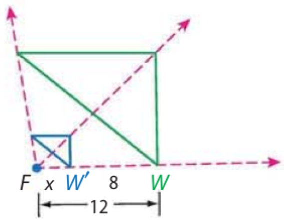
16.



17.

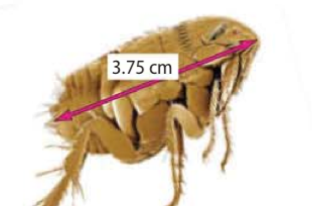


18.



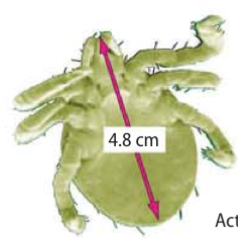
INSECTS **When viewed under a microscope, each insect has the measurement given on the picture. Given the actual measure of each insect, what magnification was used? Explain your reasoning.**

19.



Cat Flea
Actual Length: 2.5 mm

20.



Spider Mite
Actual Length: 0.5 mm

Example 3 **CCSS SENSE-MAKING** **Find the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.**

21. $J(-8, 0)$, $K(-4, 4)$, $L(-2, 0)$; $k = 0.5$

22. $S(0, 0)$, $T(-4, 0)$, $V(-8, -8)$; $k = 1.25$

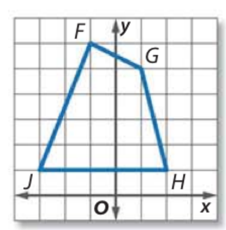
23. $A(9, 9)$, $B(3, 3)$, $C(6, 0)$; $k = \frac{1}{3}$

24. $D(4, 4)$, $F(0, 0)$, $G(8, 0)$; $k = 0.75$

25. $M(-2, 0)$, $P(0, 2)$, $Q(2, 0)$, $R(0, -2)$; $k = 2.5$

26. $W(2, 2)$, $X(2, 0)$, $Y(0, 1)$, $Z(1, 2)$; $k = 3$

27. **COORDINATE GEOMETRY** Refer to the graph of *FGHJ*.



a. Dilate *FGHJ* by a scale factor of $\frac{1}{2}$ centered at the origin, and then reflect the dilated image in the y-axis.

b. Complete the composition of transformations in part **a** in reverse order.

c. Does the order of the transformations affect the final image?

d. Will the order of a composition of a dilation and a reflection *always*, *sometimes*, or *never* affect the dilated image? Explain your reasoning.

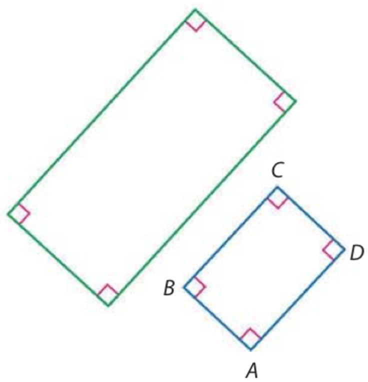
28. **PHOTOGRAPHY AND ART** To make a scale drawing of a photograph, students overlay a $\frac{1}{4}$-inch grid on a 5-inch by 7-inch high contrast photo, overlay a $\frac{1}{2}$-inch grid on a 10-inch by 14-inch piece of drawing paper, and then sketch the image in each square of the photo to the corresponding square on the drawing paper.

a. What is the scale factor of the dilation?

b. To create an image that is 10 times as large as the original, what size grids are needed?

c. What would be the area of a grid drawing of a 5-inch by 7-inch photo that used 2-inch grids?

29. **MEASUREMENT** Determine whether the image shown is a dilation of *ABCD*. Explain your reasoning.



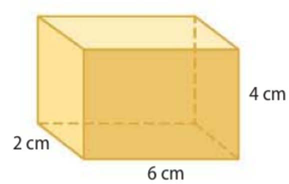
30. **COORDINATE GEOMETRY** *WXYZ* has vertices $W(6, 2)$, $X(3, 7)$, $Y(-1, 4)$, and $Z(4, -2)$.

a. Graph *WXYZ* and find the perimeter of the figure. Round to the nearest tenth.

b. Graph the image of *WXYZ* after a dilation of $\frac{1}{2}$ centered at the origin.

c. Find the perimeter of the dilated image. Round to the nearest tenth. How is the perimeter of the dilated image related to the perimeter of *WXYZ*?

31 **CHANGING DIMENSIONS** A three-dimensional figure can also undergo a dilation. Consider the rectangular prism shown.



a. Find the surface area and volume of the prism.

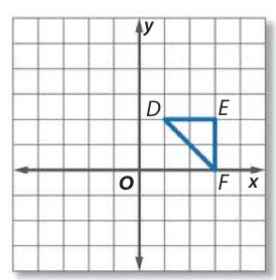
b. Find the surface area and volume of the prism after a dilation with a scale factor of 2.

c. Find the surface area and volume of the prism after a dilation with a scale factor of $\frac{1}{2}$.

d. How many times as great is the surface area and volume of the image as the preimage after each dilation?

e. Make a conjecture as to the effect a dilation with a positive scale factor *r* would have on the surface area and volume of a prism.

32. CCSS **PERSEVERANCE** Refer to the graph of $\triangle DEF$.

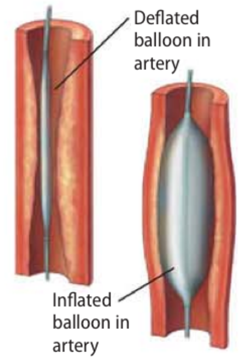


a. Graph the dilation of $\triangle DEF$ centered at point *D* with a scale factor of 3.

b. Describe the dilation as a composition of transformations including a dilation with a scale factor of 3 centered at the origin.

c. If a figure is dilated by a scale factor of 3 with a center of dilation (x, y), what composition of transformations, including a dilation with a scale factor of 3 centered at the origin, will produce the same final image?

33. **HEALTH** A coronary artery may be dilated with a balloon catheter as shown. The cross section of the middle of the balloon is a circle.



a. A surgeon inflates a balloon catheter in a patient's coronary artery, dilating the balloon from a diameter of 1.5 millimeters to 2 millimeters. Find the scale factor of this dilation.

b. Find the cross-sectional area of the balloon before and after the dilation.

Each figure shows a preimage and its image after a dilation centered at point *P*. Copy each figure, locate point *P*, and estimate the scale factor.

34.

35.

36. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate dilations centered at the origin with negative scale factors.

a. Geometric Draw $\triangle ABC$ with points $A(-2, 0)$, $B(2, -4)$, and $C(4, 2)$. Then draw the image of $\triangle ABC$ after a dilation centered at the origin with a scale factor of -2. Repeat the dilation with scale factors of $-\frac{1}{2}$ and -3. Record the coordinates for each dilation.

b. Verbal Make a conjecture about the function relationship for a dilation centered at the origin with a negative scale factor.

c. Analytical Write the function rule for a dilation centered at the origin with a scale factor of $-k$.

d. Verbal Describe a dilation centered at the origin with a negative scale factor as a composition of transformations.

H.O.T. Problems Use Higher-Order Thinking Skills

37. **CHALLENGE** Find the equation for the dilated image of the line $y = 4x - 2$ if the dilation is centered at the origin with a scale factor of 1.5.

38. **WRITING IN MATH** Are parallel lines (parallelism) and collinear points (collinearity) preserved under all transformations? Explain.

39. **CCSS ARGUMENTS** Determine whether invariant points are *sometimes*, *always*, or *never* maintained for the transformations described below. If so, describe the invariant point(s). If not, explain why invariant points are not possible.

a. dilation of *ABCD* with scale factor 1

b. rotation of $\overline{AB}$ 74° about *B*

c. reflection of $\triangle MNP$ in the *x*-axis

d. translation of *PQRS* along $\langle 7, 3 \rangle$

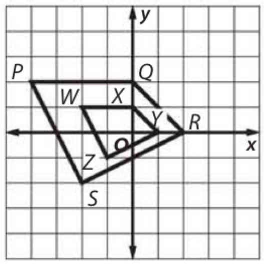
e. dilation of $\triangle XYZ$ centered at the origin with scale factor 2

40. **OPEN ENDED** Graph a triangle. Dilate the triangle so that its area is four times the area of the original triangle. State the scale factor and center of your dilation.

41. **WRITING IN MATH** Can you use transformations to create congruent figures, similar figures, and equal figures? Explain.

Standardized Test Practice

42. EXTENDED RESPONSE Quadrilateral *PQRS* was dilated to form quadrilateral *WXYZ*.



a. Is the dilation from *PQRS* to *WXYZ* an enlargement or reduction?

b. Which number *best* represents the scale factor for this dilation?

43. ALGEBRA How many ounces of pure water must a pharmacist add to 50 ounces of a 15% saline solution to make a solution that is 10% saline?

A 25 **C** 15

B 20 **D** 5

44. Tionna wants to replicate a painting in an art museum. The painting is 3 feet wide and 6 feet long. She decides on a dilation reduction factor of 0.25. What size paper should she use?

F 4 in. × 8 in. **H** 8 in. × 16 in.

G 6 in. × 12 in. **J** 10 in. × 20 in.

45. SAT/ACT For all x, $(x - 7)^2 = ?$

A $x^2 - 49$ **D** $x^2 - 14x + 49$

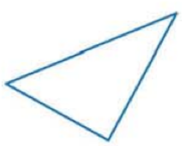
B $x^2 + 49$ **E** $x^2 + 14x - 49$

C $x^2 - 14x - 49$

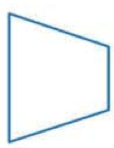
Spiral Review

State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number. (Lesson 9-5)

46.



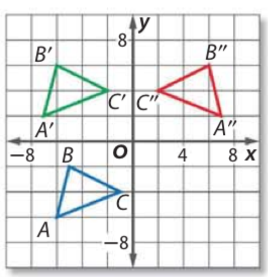
47.



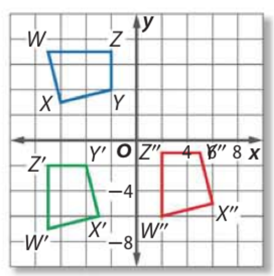
48.

Describe the transformations that combined to map each figure. (Lesson 9-4)

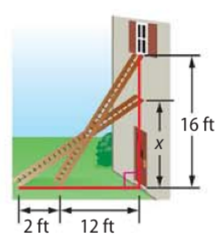
49.



50.



51. PAINTING A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbor's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach? (Lesson 8-2)



Skills Review

Find the value of *x* to the nearest tenth.

52. $58.9 = 2x$

53. $\frac{108.6}{\pi} = x$

54. $228.4 = \pi x$

55. $\frac{336.4}{x} = \pi$

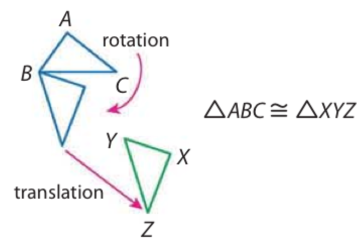
EXTEND 9-6 Geometry Lab
Establishing Triangle Congruence and Similarity

In Chapter 4, two triangles were defined to be congruent if all of their corresponding parts were congruent and the criteria for proving triangle congruence (SAS, SSS, and ASA) were presented as postulates. Triangle congruence can also be defined in terms of rigid motions (reflections, translations, rotations).

The **principle of superposition** states that two figures are congruent if and only if there is a rigid motion or a series of rigid motions that maps one figure exactly onto the other. We can use the following assumed properties of rigid motions to establish the SAS, SSS, and ASA criteria for triangle congruence.

- The distance between points is preserved. Sides are mapped to sides of the same length.
- Angle measures are preserved. Angles are mapped to angles of the same measure.

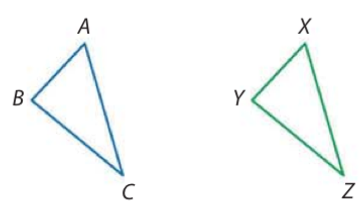
CCSS Common Core State Standards
Content Standards
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
Mathematical Practices 5



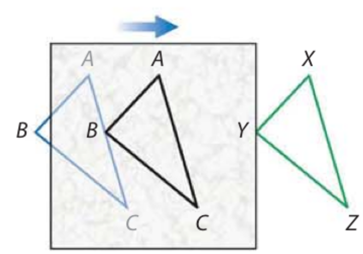
Activity 1 Establish Congruence

Use a rigid motion to map side $\overline{AB}$ of $\triangle ABC$ onto side $\overline{XY}$ of $\triangle XYZ$, $\angle A$ onto $\angle X$, and side $\overline{AC}$ onto side $\overline{XZ}$.

Step 1 Copy the triangles below onto a sheet of paper.



Step 2 Copy $\triangle ABC$ onto a sheet of tracing paper and label. Translate the paper until $\overline{AB}$, $\angle A$, and $\overline{AC}$ lie exactly on top of $\overline{XY}$, $\angle X$, and $\overline{XZ}$.



Analyze the Results

1. Use this activity to explain how the SAS criterion for triangle congruence follows from the definition of congruence in terms of rigid motions. (*Hint*: Extend lines on the tracing paper.)

2. Use the principle of superposition to explain why two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

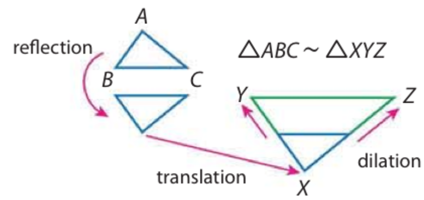
Using the same triangles shown above, describe the steps in an activity to illustrate the indicated criterion for triangle congruence. Then explain how this criterion follows from the principle of superposition.

3. SSS

4. ASA

Two figures are similar if there is a rigid motion, or a series of rigid motions, followed by a dilation, or vice versa, that map one figure exactly onto the other. We can use the following assumed properties of dilations to establish the AA criteria for triangle similarity.

- Angle measures are preserved. Angles are mapped to angles of the same measure.
- Lines are mapped to parallel lines and sides are mapped to parallel sides that are longer or shorter in the ratio given by the scale factor.



Activity 2 Establish Similarity

Use a rigid motion followed by a dilation to map $\angle B$ onto $\angle Y$ and $\angle A$ onto $\angle X$.

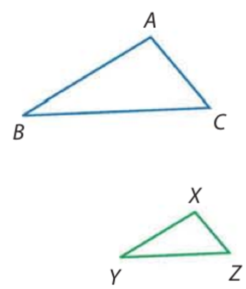
Step 1 Copy the triangles below onto a sheet of paper.

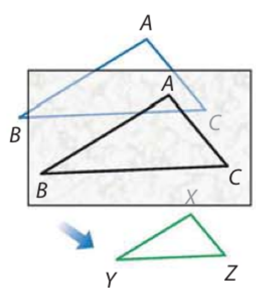
Step 2 Copy $\triangle ABC$ onto tracing paper and label.

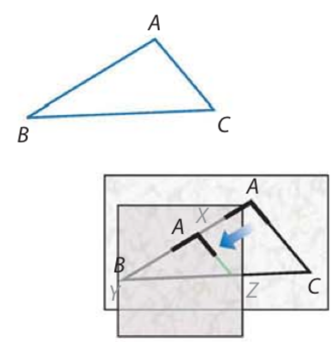
Step 3 Translate the paper until $\angle B$ lies exactly on top of $\angle Y$. Tape this paper down so that it will not move.

Step 4 On another sheet of tracing paper, copy and label $\angle A$.

Step 5 Translate this second sheet of tracing paper along the line from A to Y on the first sheet, until this second $\angle A$ lies exactly on top of $\angle X$.







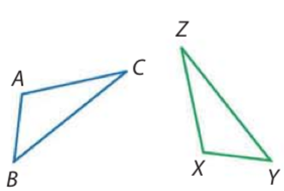
Analyze the Results

5. Use this activity to explain how the AA criterion for triangle similarity follows from the definition of similarity in terms of dilations. (*Hint*: Use parallel lines.)

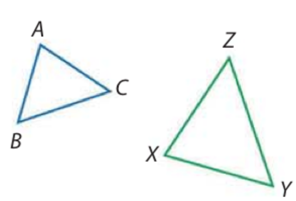
6. Use the definition of similarity in terms of transformations to explain why two triangles are similar if all corresponding pairs of angles are congruent and all corresponding pairs of sides are proportional.

Use a series of rigid motions and/or dilations to determine whether $\triangle ABC$ and $\triangle XYZ$ are *congruent, similar,* or *neither.*

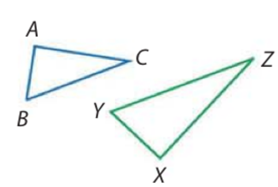
7.



8.



9.



CHAPTER 9 Study Guide and Review

Study Guide

KeyConcepts

Reflections (Lesson 9-1)

- A reflection is a transformation representing a flip of a figure over a point, line, or plane.

Translations (Lesson 9-2)

- A translation is a transformation that moves all points of a figure the same distance in the same direction.
- A translation maps each point to its image along a translation vector.

Rotations (Lesson 9-3)

- A rotation turns each point in a figure through the same angle about a fixed point.

Compositions of Transformations (Lesson 9-4)

- A translation can be represented as a composition of reflections in parallel lines and a rotation can be represented as a composition of reflections in intersecting lines.

Symmetry (Lesson 9-5)

- The line of symmetry in a figure is a line where the figure could be folded in half so that the two halves match exactly.
- The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.
- The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

Dilations (Lesson 9-6)

- Dilations enlarge or reduce figures proportionally.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.

KeyVocabulary

angle of rotation (p. 640)
axis symmetry (p. 665)
center of rotation (p. 640)
composition of transformations (p. 651)
glide reflection (p. 651)
line of reflection (p. 623)
line of symmetry (p. 663)
line symmetry (p. 663)
magnitude of symmetry (p. 664)
order of symmetry (p. 664)
plane symmetry (p. 665)
rotational symmetry (p. 664)
symmetry (p. 663)
translation vector (p. 632)

VocabularyCheck

Choose the term that best completes each sentence.

1. When a transformation is applied to a figure, and then another transformation is applied to its image, this is a(n) (composition of transformations, order of symmetries).
2. If a figure is folded across a straight line and the halves match exactly, the fold line is called the (line of reflection, line of symmetry).
3. A (dilation, glide reflection) enlarges or reduces a figure proportionally.
4. The number of times a figure maps onto itself as it rotates from 0° to 360° is called the (magnitude of symmetry, order of symmetry).
5. A (line of reflection, translation vector) is the same distance from each point of a figure and its image.
6. A figure has (a center of rotation, symmetry) if it can be mapped onto itself by a rigid motion.
7. A glide reflection includes both a reflection and a (rotation, translation).
8. To rotate a point (90°, 180°) counterclockwise about the origin, multiply the y-coordinate by −1 and then interchange the x- and y-coordinates.
9. A (vector, reflection) is a congruence transformation.
10. A figure has (plane symmetry, rotational symmetry) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

Lesson-by-Lesson Review

9-1 Reflections

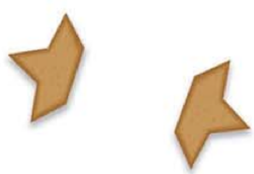
Graph each figure and its image under the given reflection.

11. rectangle *ABCD* with *A*(2, −4), *B*(4, −6), *C*(7, −3), and *D*(5, −1) in the *x*-axis

12. triangle *XYZ* with *X*(−1, 1), *Y*(−1, −2), and *Z*(3, −3) in the *y*-axis

13. quadrilateral *QRST* with *Q*(−4, −1), *R*(−1, 2), *S*(2, 2), and *T*(0, −4) in the line $y = x$

14. **ART** Anita is making the two-piece sculpture shown for a memorial garden. In her design, one piece of the sculpture is a reflection of the other, to be placed beside a sidewalk that would be located along the line of reflection. Copy the figures and draw the line of reflection.



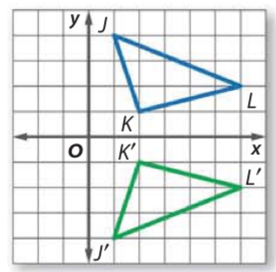
Example 1

Graph △*JKL* with vertices *J*(1, 4), *K*(2, 1), and *L*(6, 2) and its reflected image in the *x*-axis.

Multiply the *y*-coordinate of each vertex by −1.

(*x*, *y*)	→	(*x*, −*y*)
J(1, 4)	→	*J′*(1, −4)
K(2, 1)	→	*K′*(2, −1)
L(6, 2)	→	*L′*(6, −2)

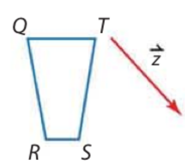
Graph △*JKL* and its image △*J′K′L′*.



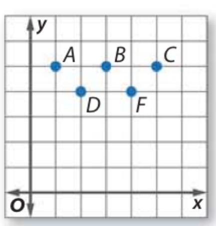
9-2 Translations

15. Graph △*ABC* with vertices *A*(0, −1), *B*(2, 0), *C*(3, −3) and its image along ⟨−5, 4⟩.

16. Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.



17. **DANCE** Five dancers are positioned onstage as shown. Dancers *B*, *F*, and *C* move along ⟨0, −2⟩, while dancer *A* moves along ⟨5, −1⟩. Draw the dancers' final positions.



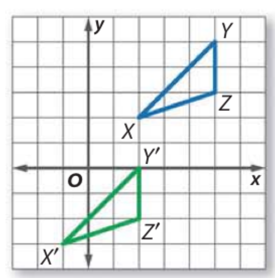
Example 2

Graph △*XYZ* with vertices *X*(2, 2), *Y*(5, 5), *Z*(5, 3) and its image along ⟨−3, −5⟩.

The vector indicates a translation 3 units left and 5 units down.

(*x*, *y*)	→	(*x* − 3, *y* − 5)
X(2, 2)	→	*X′*(−1, −3)
Y(5, 5)	→	*Y′*(2, 0)
Z(5, 3)	→	*Z′*(2, −2)

Graph △*XYZ* and its image △*X′Y′Z′*.

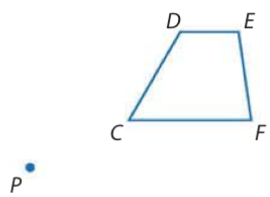


CHAPTER 9

Study Guide and Review *Continued*

9-3 Rotations

18. Copy trapezoid *CDEF* and point *P*. Then use a protractor and ruler to draw a 50° rotation of *CDEF* about point *P*.



Graph each figure and its image after the specified rotation about the origin.

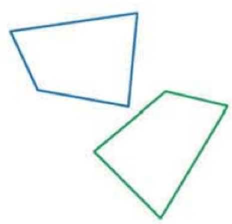
19. $\triangle MNO$ with vertices $M(-2, 2)$, $N(0, -2)$, $O(1, 0)$; 180°

20. $\triangle DGF$ with vertices $D(1, 2)$, $G(2, 3)$, $F(1, 3)$; 90°

Each figure shows a preimage and its image after a rotation about a point *P*. Copy each figure, locate point *P*, and find the angle of rotation.

21.

22.



Example 3

Triangle *ABC* has vertices $A(-4, 0)$, $B(-3, 4)$, and $C(-1, 1)$. Graph $\triangle ABC$ and its image after a rotation 270° about the origin.

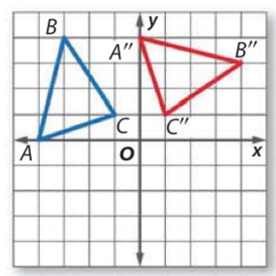
One method to solve this is to combine a 180° rotation with a 90° rotation. Multiply the *x*- and *y*-coordinates of each vertex by −1.

(x, y)	→	$(-x, -y)$
$A(-4, 0)$	→	$A'(4, 0)$
$B(-3, 4)$	→	$B'(3, -4)$
$C(-1, 1)$	→	$C'(1, -1)$

Multiply the *y*-coordinate of each vertex by −1 and interchange.

$(-x, -y)$	→	$(y, -x)$
$A'(4, 0)$	→	$A''(0, 4)$
$B'(3, -4)$	→	$B''(4, 3)$
$C'(1, -1)$	→	$C''(1, 1)$

Graph $\triangle ABC$ and its image $\triangle A''B''C''$.



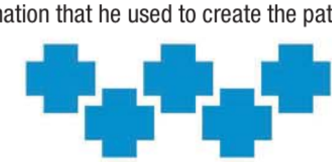
9-4 Compositions of Transformations

Graph each figure with the given vertices and its image after the indicated transformation.

23. $\overline{CD}$: $C(3, 2)$ and $D(1, 4)$
Reflection: in $y = x$
Rotation: 270° about the origin.

24. $\overline{GH}$: $G(-2, -3)$ and $H(1, 1)$
Translation: along $\langle 4, 2 \rangle$
Reflection: in the *x*-axis

25. PATTERNS Jeremy is creating a pattern for the border of a poster using a stencil. Describe the transformation combination that he used to create the pattern below.



Example 4

The endpoints of $\overline{RS}$ are $R(4, 3)$ and $S(1, 1)$. Graph $\overline{RS}$ and its image after a translation along $\langle -5, -1 \rangle$ and a rotation 180° about the origin.

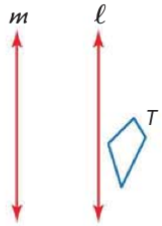
Step 1 translation along $\langle -5, -1 \rangle$

(x, y)	→	$(x - 5, y - 1)$
$R(4, 3)$	→	$R'(-1, 2)$
$S(1, 1)$	→	$S'(-4, 0)$

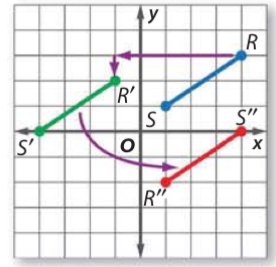
Step 2 rotation 180° about origin

(x, y)	→	$(-x, -y)$
$R'(-1, 2)$	→	$R''(1, -2)$
$S'(-4, 0)$	→	$S''(4, 0)$

26. Copy and reflect figure T in line ℓ and then line m. Then describe a single transformation that maps T onto T''.



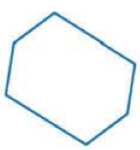
Step 3 Graph $\overline{RS}$ and its image $\overline{R''S''}$.



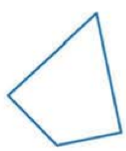
9-5 Symmetry

State whether each figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

27.



28.

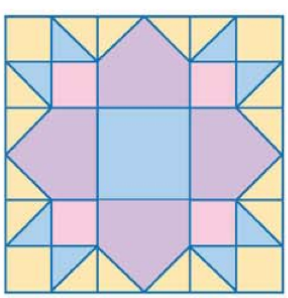


State whether each figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

29.

30.

31. **KNITTING** Amy is creating a pattern for a scarf she is knitting for her friend. How many lines of symmetry are there in the pattern?



Example 5

State whether each figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

a.

The light bulb has both plane and axis symmetry.

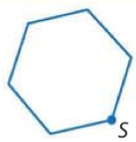
b.

The prism has plane symmetry.

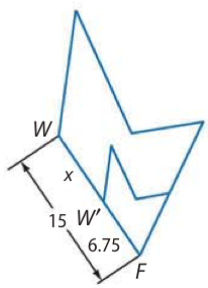
CHAPTER 9 Study Guide and Review *Continued*

9-6 Dilations

32. Copy the figure and point *S*. Then use a ruler to draw the image of the figure under a dilation with center *S* and scale factor $r = 1.25$.



33. Determine whether the dilation from figure *W* to *W′* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation and *x*.



34. **CLUBS** The members of the Math Club use an overhead projector to make a poster. If the original image was 6 inches wide, and the image on the poster is 4 feet wide, what is the scale factor of the enlargement?

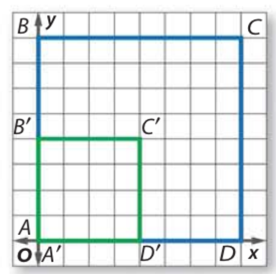
Example 6

Square *ABCD* has vertices *A*(0, 0), *B*(0, 8), *C*(8, 8), and *D*(8, 0). Find the image of *ABCD* after a dilation centered at the origin with a scale factor of 0.5.

Multiply the *x*- and *y*-coordinates of each vertex by the scale factor, 0.5.

(*x*, *y*)	→	**(0.5*x*, 0.5*y*)**
A(0, 0)	→	***A′***(0, 0)
B(0, 8)	→	***B′***(0, 4)
C(8, 8)	→	***C′***(4, 4)
D(8, 0)	→	***D′***(4, 0)

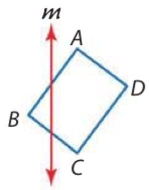
Graph *ABCD* and its image *A′B′C′D′*.



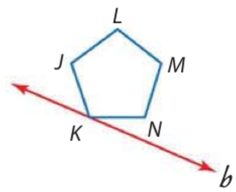
CHAPTER 9 Practice Test

Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

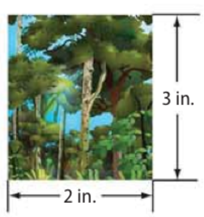
1.



2.

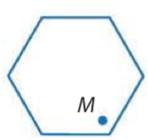


3. **PROJECTS** Eduardo wants to enlarge the picture below to 4 inches by 6 inches for a school project. If his school's copy machine can only enlarge up to 150% by whole number percents, find two whole number percents by which he can enlarge the piece and get as close to 4 inches by 6 inches or less.

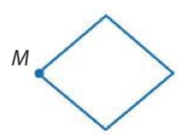


Copy the figure and point *M*. Then use a ruler to draw the image of the figure under a dilation with center *M* and the scale factor *r* indicated.

4. $r = 1.5$



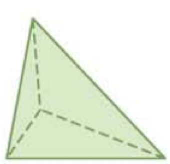
5. $r = \frac{1}{3}$



6. **PARKS** Isabel is on a ride at an amusement park that slides the rider to the right, and then rotates counterclockwise about its own center 60° every 2 seconds. How many seconds pass before Isabel completes one full rotation?

State whether each figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

7.



8.

Graph each figure and its image under the given transformation.

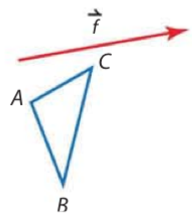
9. $\square FGHJ$ with vertices $F(-1, -1)$, $G(-2, -4)$, $H(1, -4)$, and $J(2, -1)$ in the x-axis

10. $\triangle ABC$ with vertices $A(0, -1)$, $B(2, 0)$, $C(3, -3)$; $\langle -5, 4 \rangle$

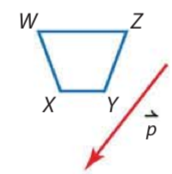
11. quadrilateral $WXYZ$ with vertices $W(2, 3)$, $X(1, 1)$, $Y(3, 0)$, $Z(5, 2)$; 180° about the origin

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

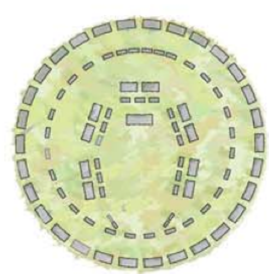
12.



13.



14. **ART** An artist's rendition of what Stonehenge, a famous archeological site in England, would have looked like before the stones fell or were removed, is shown below. What is the order and magnitude of symmetry for the outer ring?



15. **MULTPLE CHOICE** What transformation or combination of transformations does the figure below represent?

A dilation

B glide reflection

C rotation

D translation

CHAPTER 9

Preparing for Standardized Tests

Work Backward

In most problems, a set of conditions or facts is given and you must find the end result. However, some problems give you the end result and ask you to find something that happened earlier in the process. To solve problems like this, you must work backward.

Strategies for Working Backward

Step 1

Look for keywords that indicate you will need to work backward to solve the problem.

Sample Keywords:

- What was the **original**…?
- What was the value **before**…?
- Where was the **starting** or **beginning**…?

Step 2

Undo the steps given in the problem statement to solve.

- List the sequence of steps from the beginning to the end result.
- Begin with the end result. Retrace the steps in reverse order.
- "Undo" each step using inverses to get back to the original value.

Step 3

Check your solution if time permits.

- Make sure your answer makes sense.
- Begin with your answer and follow the steps in the problem statement forward to see if you get the same end result.

Standardized Test Example

Solve the problem below. Responses will be graded using the short-response scoring rubric shown.

Kelly is using a geometry software program to experiment with transformations on the coordinate grid. She began with a point and translated it 4 units up and 8 units left. Then she reflected the image in the x-axis. Finally, she dilated this new image by a scale factor of 0.5 with respect to the origin to arrive at $(-1, -4)$. What were the original coordinates of the point?

Scoring Rubric	
Criteria	**Score**
Full Credit: The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit: • The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1
No Credit: Either an answer is not provided or the answer does not make sense.	0

Read the problem statement carefully. You are given a sequence of transformations of a point on a coordinate grid. You know the coordinates of the final image and are asked to find the original coordinates. Undo each transformation in reverse order to work backward and solve the problem.

Example of a 2-point response:

original point → translation → reflection → dilation → end result

Begin with the coordinates of the end result and work backward.

Dilate by 2 to undo the dilation by 0.5:

$(-1, -4) \rightarrow (-1 \times 2, -4 \times 2) = (-2, -8)$

Reflect back across the x-axis to undo the reflection:

$(-2, -8) \rightarrow (-2, 8)$

Translate 4 units down and 8 units right to undo the translation:

$(-2, 8) \rightarrow (-2 + 8, 8 - 4) = (6, 4)$

The original coordinates of the point were (6, 4).

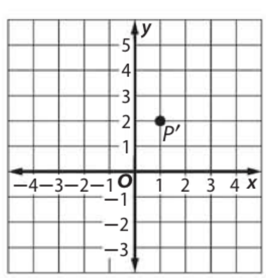
The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

Exercises

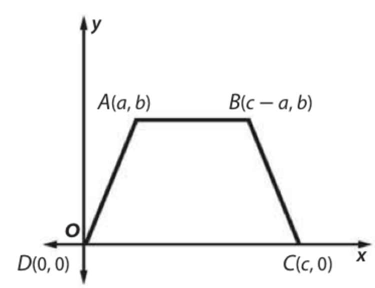
Solve each problem. Show your work. Responses will be graded using the short-response scoring rubric given at the beginning of the lesson.

1. A flea landed on a coordinate grid. The flea hopped across the x-axis and then across the y-axis in the form of two consecutive reflections. Then it walked 9 units to the right and 4 units down. If the flea's final position was at (4, −1), what point did it originally land on?

2. The coordinate grid below shows the final image when a point was rotated 90° clockwise about the origin, dilated by a scale factor of 2, and shifted 7 units right. What were the original coordinates?



3. Figure $ABCD$ is an isosceles trapezoid.



Which of the following are the coordinates of an endpoint of the median of $ABCD$?

A $\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$

B $\left(\frac{2c-a}{2}, \frac{b}{2}\right)$

C $\left(\frac{c}{2}, 0\right)$

D $\left(\frac{c}{2}, b\right)$

4. If the measure of an interior angle of a regular polygon is 108, what type of polygon is it?

F octagon

G hexagon

H pentagon

J triangle

CHAPTER 9 Standardized Test Practice

Cumulative, Chapters 1 through 9

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Point N has coordinates $(4, -3)$. What will the coordinates of its image be after a reflection across the y-axis?

A $N'(-3, 4)$

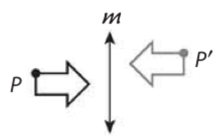
B $N'(-4, 3)$

C $N'(4, 3)$

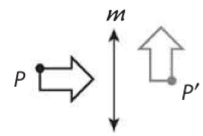
D $N'(-4, -3)$

2. Which pair of figures shows a reflection across the line followed by a translation up?

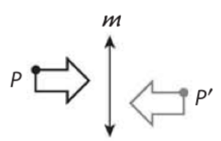
F



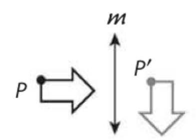
H



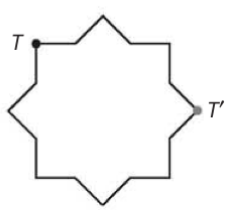
G



J



3. What is the angle of clockwise rotation that maps point T onto T' in the figure below?



A 90°

B 120°

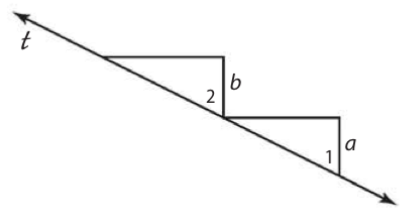
C 135°

D 145°

Test-Taking Tip

Question 3 How many points are there on the star? Divide 360° by this number to find the angle of rotation from one point to the next.

4. **Given**: $a \parallel b$



Which statement below justifies the conclusion that $\angle 1 \cong \angle 2$?

F If $a \parallel b$ and are cut by transversal t, then alternate exterior angles are congruent.

G If $a \parallel b$ and are cut by transversal t, then alternate interior angles are congruent.

H If $a \parallel b$ and are cut by transversal t, then corresponding angles are congruent.

J If $a \parallel b$ and are cut by transversal t, then vertical angles are congruent.

5. What is the geometric mean of 8 and 18?

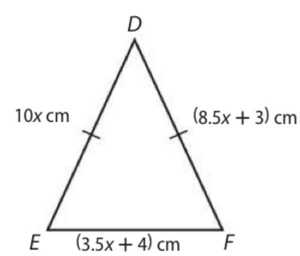
A 9

B 10

C 11

D 12

6. Which of the following is a side length in isosceles triangle DEF?



F 2 cm

G 8 cm

H 9 cm

J 11 cm

7. Which of the following has exactly two pairs of consecutive congruent sides?

A kite

B parallelogram

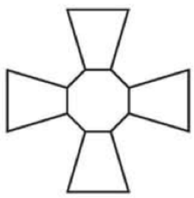
C rhombus

D trapezoid

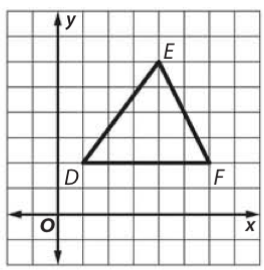
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. State whether the figure has rotational symmetry. If so, copy the figure, locate the center, and state the order and magnitude of symmetry.



9. Dilate the figure shown on the coordinate grid by a scale factor of 1.5 centered at the origin.

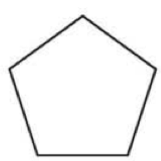


10. Complete the following statement.

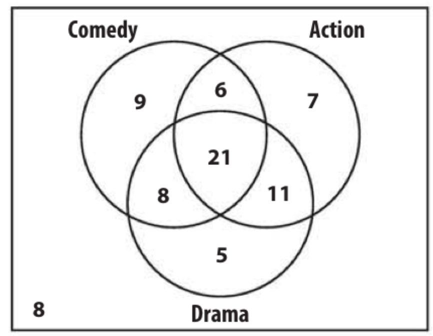
According to the Angle Bisector Theorem, if a point is on the bisector of an angle, then it is _______ .

11. Regina left her office downtown and traveled 3 blocks west and 5 blocks north. Write a translation vector to describe her route.

12. What is the interior angle measure of the regular pentagon?



13. **GRIDDED RESPONSE** A group of 75 students were asked what types of movies they like to watch. The results are shown in the Venn diagram.



How many students said that they like to watch action and drama movies, but not comedy?

Extended Response

Record your answers on a sheet of paper. Show your work.

14. Rodrigo is making a scale model.

a. The actual length of the Golden Gate Bridge is about 9000 feet. If Rodrigo's model is 45 inches, what is the scale of his model?

b. How wide will Rodrigo's model of the bridge be if the actual width is 90 feet?

c. In Rodrigo's model, the tower will be 2.5 inches above the roadway. How high above the roadway is the actual tower?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Go to Lesson...	9-1	9-4	9-3	3-2	8-1	4-6	6-6	9-5	9-6	5-1	9-2	6-1	2-2	7-7